Problem-Cen Mathematics Teaching

SARAH THEULE LUBIENSKI

THROUGH MY INTERACTIONS WITH preservice and in-service teachers in California, Michigan, and New York, I have heard a variety of perspectives on instruction in problem-centered mathematics instruction. I have watched educators struggle to find a role for problem solving in their classes. What constitutes teaching through problem solving? Exploring answers to this question can raise awareness of one's own perspectives and lead to deeper knowledge of problem-centered mathematics teaching.

Three teachers search for understanding of problem-centered instruction

To that end, let us listen in on a hypothetical—albeit based on compilations of real conversations and events—discussion involving three teachers who are searching for understanding of problem-centered instruction.

SARAH LUBIENSKI, *lubienst@buffalostate.edu, teaches at Buffalo State College, Buffalo, NY 14222. Her interest in differing interpretations of problem-centered learning grows out of her research with the Education Policy and Practice Study housed at Michigan State University.*

tered

Meet Anne, Ben, and Cara

ANNE, BEN, AND CARA ARE EXPERIENCED SIXTH-grade teachers from different urban schools, participating in a two-week professional development institute. At the beginning of the institute, a mathematics education professor talked through several overhead transparencies outlining key points of the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) and the *Professional Standards for Teaching Mathematics* (NCTM 1991). The professor emphasized the need for mathematics classrooms to involve students in problem solving, communicating, and reasoning. She then had participants solve the Corral Problem (see **fig. 1**) in groups of three. Anne, Ben, and Cara worked together as a group. After the groups had finished, the professor asked the participants what concepts students could learn from the problem; they listed counting, multiplication, area, perimeter, rectangles, squares, business mathematics, statistics, and reasoning. Little discussion took place re-

Rosa needs your help designing a corral for her horses. Rosa has looked at lots of designs, and has decided two things:

1. She wants a corral that is in the shape of a rectangle.

2. She wants the corral to give her horse the largest possible area.

Rosa has 16 units of fence to use. The sides of the corral must be made up of whole units of fence (for example, a side cannot be 2½ units long).

1. On dot or graph paper, show all the four-sided corral designs that Rosa can make with 16 units of fence. Label them so that Rosa can see which one has the largest area.

2. Suppose that Rosa decided to use 24 units of fence. What designs would be possible? Which one would have the largest area?

3. Is there a general rule for figuring out which type of design will give the largest area for any length of fence?

Adapted from the *New Standards Reference Examination, Pilot Edition* (New Standards Project 1995).

Fig. 1 The Corral Problem

garding exactly what or how students would learn about these concepts or which concepts were truly at the heart of the problem and which others were peripheral.

In the second session, a teacher-leader introduced an origami art video, and participants followed step-by-step instructions given in the video as they made origami birds. After the video, the leader of the session explained that although mathematical language was rarely used in the video, several mathematical ideas were still involved in making the origami figures. She asked participants to think of the mathematical ideas to which students could be exposed through origami; and they listed parallel lines, fractions, triangles, squares, and other topics. At the end of this activity, Cara voiced her growing frustration with her experiences thus far: "We have done several interesting activities, and we have tended to complete each one by listing mathematical ideas students can learn from these activities. Yet, we haven't talked about *how* this should really happen." The leader said that all teachers struggle with this issue and that each teacher needs to figure out what approach is most effective for her or his particular students.

"Students learn best by making sense of things themselves"

Cara was not satisfied with this answer. Ben and Anne told Cara that they were intrigued with the question that she had raised, so the three decided to have lunch together to discuss it.

The Lunchtime Discussion

CARA. ANNE, WHAT HAVE YOU BEEN DOING SO far to incorporate problem solving into your teaching?

Anne. I have been trying to include more problem solving in my mathematics classes because my students like it, and I want to make sure students know how to apply the mathematics they learn. The new textbook our school just adopted seems pretty traditional, but it claims to be compatible with the NCTM Standards. The book begins with a chapter on problem solving, and I go over the different problem-solving strategies with my students so that they have tools to solve problems in our class and in real life. The rest of the chapters are pretty much the same as they are in our old books. At the end of each chapter, there are a few story problems that involve applying the skills learned in the chapter. I assign these problems to my students whenever possible. I have also begun to collect puzzles, games, and other problem-solving activities, and I sometimes use these for students who get done with their work early. I feel bad that I haven't had time to find more good problems, but I don't know when I could fit them in, anyway. I don't know what I can remove from our curriculum to include more problem solving.

Ben. Why do you think you have to throw things out of the curriculum to do more problem solving?

Anne. Well, something's got to go.

Ben. Not necessarily. I think NCTM is pushing for problem solving to be a *means* of learning math—not just a way of applying it. I have gone to many workshops over the past few years, and I have collected lots of good problems. Instead of trying to supplement our school's old textbook, I have tried to cover the same ideas the book does, but I do it by giving my students interesting problems to solve. Most of my students really get into the problems—they don't even know we're doing mathematics half the time.

Cara. But how are they learning mathematics if they don't know they are?

Ben. Well, like with the origami [we did] today. My students could make an origami figure and have to use lots of mathematical ideas, but they don't necessarily know they're learning mathematics.

Cara. But that is what I was getting at with my question after the origami. How to use problem solving as a way of teaching mathematics is a complicated issue that I've been thinking a lot about lately, [but] it seems to get glossed over in these settings. I'm not convinced that just because we give students a problem to solve, that students are going to learn the mathematical ideas that we think they are. I have been to several workshops now where we do this "listing" of mathematical ideas "embedded" in problems, and it's a good exercise for thinking about mathematics. But it seems like the implication is that students will somehow naturally abstract these ideas from each problem I give them, and I'm not sure this is accurate—especially not for all of my students. Not to mention that I think students need help in understanding how these ideas fit within the larger mathematical terrain.

Anne. I see what you mean. I guess that's why I feel better talking through ideas with my students and having them practice each individual skill and idea *first* to make sure they are "getting" what they are supposed to. Then I have my students *apply* these ideas and skills in real-world problems so they learn how to use their mathematical knowledge.

Cara. Yet, I have seen problem solving used as a powerful way of *learning* new concepts and skills. I believe students learn best by making sense of things themselves instead of having the informa-

tion handed to them. Sometimes I do need to give students information they need to work on a mathematics problem (such as what a corral is, in the case of the Corral Problem), but once my students have the needed background information, I think it's fruitful for them to struggle a bit with the mathematics in a problem. I just want to be sure that after the struggle, students really do learn the intended, "big" ideas and that I as a teacher am being as effective as possible at helping this happen. In my experience, the students who have the most difficulty learning mathematics through problem solving do so because they focus more on a problem's real-world story line than [on] the mathematical ideas I intend. I feel it's my job to make sure students do not wander fruitlessly or focus on the problem's context in ways that allow them to miss the mathematical point.

Ben. I guess I tend to be more "laid back," but I suppose some of my students could be missing important ideas occasionally. After our problem-solving activities, there is always a time when students talk about how they solved the problems, and this is usually when some of the mathematical ideas are mentioned; but I tend to assume that as the teacher I shouldn't harp on the specific mathematical ideas because then it will become too much of a traditional mathematics lesson. I want my class to be fun—not a boring mathematics lecture.

Cara. And I, too, tend to have my students report back after problem-solving activities, but I do "harp" on the mathematics at that point. Not that I am the one doing all the talking, but I have a very clear agenda in my mind. Before I teach, I plan all the questions I might want to ask to pull out the "big" ideas from students and get them on the table for all of us to discuss. But you are right in that I have to struggle with how much I tell students and how much I try to push or pull out from them. I tend to ask guiding questions to prompt students to think about key ideas, but I worry that these questions could be hints that some students get and others—especially those whose families have different communication styles than mine—won't get. Something Anne said earlier about explicitly teaching students problem-solving skills has made me think about what more I could be doing to help my students learn norms for understanding mathematics through problem solving.

Anne. You two are talking about problem solving differently than I have thought much about [it] before. I want to make this discussion more concrete by talking about the Corral Problem that we did today. I like the problem because it shows how area and perimeter are used in the real world. If I were to use the problem, I would give it to students at the end of our unit on area and perimeter—probably as a homework problem. At that point, my students would have learned how to find the area and perimeter of rectangles, and this problem would give them the chance to see how these ideas can be applied. How would you use the problem?

Ben. I would give the problem to students as a way of introducing area and perimeter. The class would solve it in small groups of three or four students each. When they were done, each group would report to the class so we could compare answers to each question and talk about how the students got their answers.

Cara. Exactly what would your students learn about perimeter and area, Ben?

Ben. I guess students who had never heard of area or perimeter would learn what the terms mean and, through working on the problem, students could find a method for calculating areas and perimeters of rectangles.

Cara. When we were looking at the problem, I saw that the first two questions asked students to calculate areas and perimeters; and I agree with you, Ben, that these questions could help students learn about these ideas and the distinctions between the two. Yet I think the main idea of the problem as a whole is the relationship among shape, area, and perimeter; that is, given a fixed perimeter, the more "square-shaped" the rectangle is, the larger the area. Ben, like you, I would give this problem to my students to solve in small groups, and then we would have a follow-up discussion. In that discussion, I would want to go beyond having my students report their answers and ways of finding the areas. I would plan a series of questions designed to prompt my students to think about the relationship between the first two questions posed in the problem and the overall pattern relating shape and area. Also, I noticed that the problem doesn't ask why we should expect this pattern to always hold. We didn't get time to discuss this [omission] during the session this morning, but I have been thinking about why this makes sense. [She begins drawing on a napkin—see **fig. 2.**] In the long, thin, 7×1 arrangement, each little square of area is surrounded by "fence," but in a 4×4 arrangement, the fence "traps" more area without touching all the squares. Both during the groups' explorations and during the class discussion that follows, I would ask my students questions to get them to think about these relationships. ☞

"I make sure students do not miss the mathematical point"

Ben. I didn't notice all those ideas in the problem, but I see them now. If *I* didn't notice them when I solved it, then I guess many students probably wouldn't notice them either. I still think students could get a lot out of the problem without pushing all the ideas you talked about, but I must admit that they would probably learn more mathematics if I went into my teaching with a more careful plan for pulling out key ideas. Yet, don't you worry about making lessons too boring or teacher centered? What about the possibility that students could see something important that was not part of your agenda? I think of NCTM's Standards as giving students a larger role in shaping the agenda.

Cara. That's a good point. I certainly want students to get a sense of mathematical problem solving as something more than just guessing what the teacher is going for. I want them to see it as a creative process, and I want students to see themselves as capable of coming up with interesting ideas I had not thought of. I guess I need to [be flexible in] my agenda when a student pushes in another direction that seems promising.

Anne. Cara, you sure saw a lot in that problem that I never saw. I don't know how you can analyze all problems so carefully and then put them together to form a curriculum. Is that what you do?

Cara. Finding time to plan carefully is one of my biggest struggles. Luckily, I don't plan alone. A few years ago, a couple of other teachers from my school and I decided to plan our mathematics lessons together and to have in-depth discussions of the mathematics we were teaching. Through this [approach], as well as through courses I took in my graduate program, I have become more confident that I can make sense of mathematical ideas for myself. I no longer accept any teaching idea or mathematics problem for its surface features, but instead, I analyze underlying mathematical and pedagogical issues. Yet, I still struggle with how to juggle using our school's traditional textbook and using more open-ended problems from other sources. Thankfully, my school just adopted a new textbook series to help implement the NCTM Standards, and we will begin using these textbooks next year. This textbook uses problems, like the Corral Problem, as its primary means of teaching mathematical ideas and skills. Still, I know this [textbook] won't resolve all my struggles—I will still wonder how much to tell students, when to lead, and when to follow. That is why I will keep looking for others, such as you two, who I can talk to and learn from.

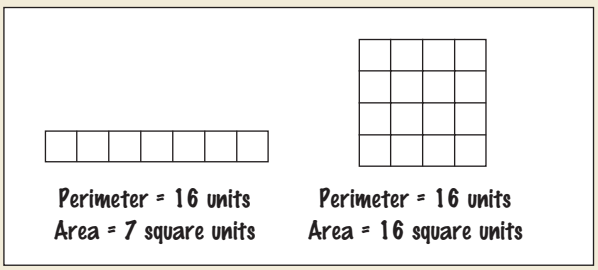

Fig. 2 Cara's drawing of two rectangles with the same perimeter

Our Discussion of the Issues

ALTHOUGH THESE TEACHERS HAVE MUCH MORE to talk about, we will pull back from their conversation now and take stock of what we have heard. One could argue that Anne, Ben, and Cara all teach in a problem-centered way, yet their styles and goals are quite different.

Some of their differences correspond with distinctions that Schroeder and Lester (1989) make between teaching *about* mathematical problem solving and teaching mathematics *for* or *through* problem solving. In these terms, Anne teaches *about* problem solving when she focuses on helping students learn specific problem-solving strategies and skills apart from other mathematical ideas. She also teaches mathematics *for* problem solving, so that her students learn key mathematical ideas and skills that they can later apply in problem situations. Anne's views differ from those who argue that problem solving should not be isolated but rather should serve as a primary *means* for learning mathematical concepts and skills. In other words,

> [Problem solving is] a method of inquiry and application, interwoven throughout the *Standards* to provide a consistent context for learning and applying mathematics. Problem situations can establish a "need to know" and foster the motivation for the development of concepts (NCTM 1989, 75).

Both Ben and Cara aspire to teach mathematics *through* problem solving, but their perspectives on this approach differ. Ben tends to assume that students will naturally learn important mathematical ideas through their experiences with interesting problems. He wants students to make sense of mathematics for themselves and, as the teacher, he wants to move away from being "the sole authority for right answers" (NCTM 1991, 3). What Ben seems to miss is the active teacher role, promoted by the Standards, which involves careful planning for, involvement in, and assessment of students' learning. Although some students might always learn what Ben expects from the problems he assigns, others are likely to miss the intended mathematical ideas (Lubienski, submitted for publication).

In contrast, Cara carefully analyzes problems and the mathematics involved, and she worries that

some students might miss key ideas if the problems are assumed to "carry the math." She enters each problem-centered lesson with a clear agenda for her students' learning and a detailed plan for guiding both her students' thinking about the problems and their reflections on the mathematical ideas that arise. Some, like Ben, might say that Cara tells her students too much in her attempts to ensure that all her students "get" what she intends from their problem-solving efforts. Others, like Anne, might say that Cara does not tell her students enough—that she should tell her students "the rules" instead of letting them try to figure them out for themselves. Cara struggles to balance her role as someone who facilitates instead of dictates. She knows from experience that students will not always see what she expects them to see when exploring a given problem. She wants to take students' discoveries seriously while ensuring that her students ultimately learn what she intends and do not wander too much in the process. This outcome is not easily attained.

Although some teachers might become discouraged when their methods of teaching are fraught with such uncertainties, Cara seeks a deeper understanding to help her manage dilemmas thoughtfully (Lampert 1985). Teaching through problem solving is complex, and teachers need to think carefully about the problems they use, the mathematics in those problems, and the ways in which students might or might not learn from them. Cara's comments point to some avenues of teacher support in using these methods. Planning and discussing mathematics with colleagues can help teachers think carefully about how to guide students' learning through specific problems. Professional development opportunities offer exposure to current ideas, as well as valuable teacher networks. Graduate classes can offer a chance to dig into complex mathematical and pedagogical issues that are often glossed over in workshop settings. Additionally, the recently published middle-grades-curriculum projects funded by the National Science Foundation offer teachers problem-centered curricula as a foundation for instruction and a common ground for discussion with other teachers.

Problem-centered instruction can involve difficult, sometimes conflicting, roles for teachers. Clarifying our interpretations of problem-centered instruction can increase our understanding of these roles and help us educate all our students more effectively. This article is intended to help readers reflect on how they see problem solving in their classrooms and to reconsider ways in which they use problems in instruction.

References

Lampert, Magdalene. "How Do Teachers Manage to Teach?" *Harvard Educational Review* 55 (1985): 178–94.

Lubienski, Sarah T. "Problem Solving as a Means toward 'Mathematics for All': A Look through a Class Lens." Submitted for publication to the *Journal for Research in Mathematics Education*.

National Council of Teachers of Mathematics (NCTM). *Curriculum and Evaluation Standards for School Mathematics.* Reston, Va.: NCTM, 1989.

———. *Professional Standards for Teaching Mathematics.* Reston, Va.: NCTM, 1991.

New Standards Project. *New Standards Reference Examination, Pilot Edition.* Pittsburgh, Pa.: Learning Research and Development Center, University of Pittsburgh, 1995.

Schroeder, Thomas L., and Frank K. Lester. "Developing Understanding in Mathematics via Problem Solving." In *New Directions for Elementary School Mathematics, Yearbook of the National Council of Teachers of Mathematics,* 31–42. Reston, Va.: NCTM, 1989.

Mathematics Teaching in the Middle School (MTMS) is interested in receiving manuscripts on the following topics. Over 70 percent of recently surveyed members indicated that these topics most concern them:

- Algebraic thinking and algebra for all
- Mathematical reasoning
- Problem solving

If you have interesting ideas or classroom-tested approaches surrounding the big ideas for any of these three topics, please share them by writing a manuscript for the journal.

Submit five copies of such a manuscript to *Mathematics Teaching in the Middle School,* NCTM, 1906 Association Drive, Reston, VA 20191-9988. If you would like hints on how to prepare a manuscript for publication, request our "*MTMS* Writer's Packet" from NCTM at the same address or send e-mail to Kathleen Lay at klay@nctm.org.