**Visual Models:**
**Multiplication and Division**
**Grade 3 – Grade 5**

<table>
<thead>
<tr>
<th>Grade 3</th>
<th>Grade 4</th>
<th>Grade 5</th>
<th>Grade 5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Equal Groups</strong></td>
<td><strong>Fraction x Whole Number</strong></td>
<td><strong>Fraction ÷ Whole number</strong></td>
<td><strong>Fraction x Whole Number</strong></td>
</tr>
<tr>
<td>6+6+6+6+6+6+6+6 = 48 8 groups of 6 = 48</td>
<td>( \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{10}{3} ) 5 groups of ( \frac{2}{3} = \frac{10}{3} )</td>
<td>( \frac{10}{3} \div 5 = \frac{2}{3} ) ( \frac{10}{3} \div 2/3 = 5 )</td>
<td>( \frac{2}{3} \times 5 = \frac{10}{3} )</td>
</tr>
<tr>
<td>6 x 8 = 48 48 ÷ 6 = 8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 x 6 = 48 48 ÷ 8 = 6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Tape Diagram</strong></th>
<th><strong>Multiplicative Comparison</strong></th>
<th><strong>Fraction x Whole Number</strong></th>
<th><strong>Fraction ÷ Whole Number</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>6+6+6+6+6+6+6+6+6+6 = 48</td>
<td>48 is 8 times as large as 6</td>
<td>( \frac{2}{3} \times 5 = \frac{10}{3} )</td>
<td>( \frac{2}{3} \times 5 = \frac{10}{3} )</td>
</tr>
<tr>
<td>6 x 8 = 48 48 ÷ 6 = 8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 x 6 = 48 48 ÷ 8 = 6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Equations in *italics* are part of the “fact family” for the model shown, so students may be able solve them using this information. However based on the CCSS-M, they are beyond the indicated grade level expectations.

This chart shows some examples of how visual models may be used, and is not an exhaustive list.

*Updated October 24, 2016*
### Visual Models: Multiplication and Division

#### Grade 3 – Grade 5

<table>
<thead>
<tr>
<th>Open Number Line</th>
<th>Grade 3</th>
<th>Grade 4</th>
<th>Grade 5</th>
<th>Grade 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>+6 +6 +6 +6 +6 +6 +6 +6</td>
<td>6+6+6+6+6+6+6+6 = 48</td>
<td>+3/5 +3/5 +3/5 +3/5 +3/5</td>
<td>10/3 ÷ 5 = 2/3</td>
<td>0.4+0.4+0.4+0.4+0.4+0.4+0.4+0.4+0.4+0.4 = 2.4</td>
</tr>
<tr>
<td>-6 -6 -6 -6 -6 -6 -6 -6</td>
<td>48-6-6-6-6-6-6-6-6 = 0</td>
<td>-3/5 -3/5 -3/5 -3/5 -3/5</td>
<td>10/3 ÷ 2/3 = 5</td>
<td>6 groups of 0.4 = 2.4</td>
</tr>
<tr>
<td>6+6+6+6+6+6+6+6 = 48</td>
<td>8 groups of 6 = 48</td>
<td>6 x 0.4 = 2.4</td>
<td>6 x 0.4 = 2.4</td>
<td>2.4 ÷ 6 = 0.4</td>
</tr>
<tr>
<td>8 x 6 = 48</td>
<td>48 ÷ 8 = 6</td>
<td>0.4 x 6 = 2.4</td>
<td>0.4 x 6 = 2.4</td>
<td>2.4 ÷ 0.4 = 6</td>
</tr>
</tbody>
</table>

### Fraction x Whole Number

\[
\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{10}{3}
\]

5 groups of \(\frac{2}{3} = \frac{10}{3}\)

\[
5 \times \frac{2}{3} = \frac{10}{3}
\]

\[
\frac{2}{3} \times 5 = \frac{10}{3}
\]

### Fraction ÷ Whole Number

\[
\frac{10}{3} ÷ 5 = \frac{2}{3}
\]

\[
10/3 ÷ 2/3 = 5
\]

\[
\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}
\]

\[
\frac{3}{8} ÷ 1/2 = 3/4
\]

\[
\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}
\]

\[
3/8 ÷ 3/4 = 1/2
\]

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Visual Models:
Multiplication and Division
Grade 3 – Grade 5

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<th>Grade 4</th>
<th>Grade 5</th>
<th>Grade 5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Array</strong></td>
<td><strong>Bar Model</strong></td>
<td><strong>Grid</strong></td>
<td><strong>Grid</strong></td>
</tr>
<tr>
<td>6 x 8 = 48</td>
<td>48 ÷ 6 = 8</td>
<td></td>
<td>0.3 x 0.4 = 0.12</td>
</tr>
<tr>
<td>8 x 6 = 48</td>
<td>48 ÷ 8 = 6</td>
<td></td>
<td>0.12 ÷ 0.3 = 0.4</td>
</tr>
</tbody>
</table>

**2-digit x 2-digit Whole Numbers**
11 x 13 = 143
13 x 11 = 143

**2-digit Divisor Whole Numbers**
143 ÷ 13 = 11
143 ÷ 11 = 13

**Area Model**

**Generic**
4 x 13 = 52
13 x 4 = 52

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Updated October 24, 2016
Visual Models:
Multiplication and Division
Grade 3 – Grade 5

Connection to Algorithms: Multiplication (3 x 1 digit)

<table>
<thead>
<tr>
<th>Method A:</th>
<th>Method B:</th>
<th>Method C:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left to right showing the partial products</td>
<td>Right to left showing the partial products</td>
<td>Right to left recording the carries below</td>
</tr>
<tr>
<td>549</td>
<td>549</td>
<td>549</td>
</tr>
<tr>
<td>× 8</td>
<td>× 8</td>
<td>× 8</td>
</tr>
<tr>
<td>4000</td>
<td>72</td>
<td>4392</td>
</tr>
<tr>
<td>8 x 5 hundreds</td>
<td>thinking:</td>
<td>thinking:</td>
</tr>
<tr>
<td>320</td>
<td>320</td>
<td>8 x 4 tens</td>
</tr>
<tr>
<td>72</td>
<td>8 x 9</td>
<td>8 x 9</td>
</tr>
<tr>
<td>4392</td>
<td>320</td>
<td>4392</td>
</tr>
<tr>
<td>8 x 5 hundreds</td>
<td></td>
<td>8 x 5 hundreds</td>
</tr>
</tbody>
</table>

Method A proceeds from left to right, and the others from right to left. In Method C, the digits representing new units are written below the line rather than above 549, thus keeping the digits of the products close to each other, e.g., the 7 from 8 x 9 = 72 is written diagonally to the left of the 2 rather than above the 4 in 549.

https://www.mathedleadership.org/docs/resources/journals/NCSMJournal_ST_Algorithms_Fuson_Beckmann.pdf
### Visual Models: Multiplication and Division

**Grade 3 – Grade 5**

**Connection to Algorithms: Multiplication (2 x 2 digit)**

**Array/area drawing for 36 x 94**

![Diagram](chart.png)

**Method D:**

- Showing the partial products
- $30 \times 90 = 2700$
- $30 \times 4 = 120$
- $6 \times 90 = 540$
- $6 \times 4 = 24$

**Method E:**

- Recording the carries below for correct place value placement
- $94 \times 36$
- $24$ (thinking: $6 \times 4$
- $540$
- $120$
- $2700$

**Method F:**

- A misleading abbreviated method
- From $30 \times 4 = 120$, the 1 is 1 hundred, not 1 ten.

**Method G: Helping Steps**

- $94 = 90 + 4$
- $\times 36 = 30 + 6$
- $30 \times 90 = 2700$
- $30 \times 4 = 120$
- $6 \times 90 = 540$
- $6 \times 4 = 24$


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### Visual Models:
#### Multiplication and Division
##### Grade 3 – Grade 5

**Connection to Algorithms: Division**

#### Area/array drawing for 966 ÷ 7

<table>
<thead>
<tr>
<th>? hundreds</th>
<th>? tens</th>
<th>? ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td></td>
<td>966</td>
</tr>
</tbody>
</table>

Thinking: A rectangle has area 966 and one side of length 7. Find the unknown side length. Find hundreds first, then tens, then ones.

\[
966 = 7 \times 100 + 7 \times 30 + 7 \times 8 \\
= 7 \times (100 + 30 + 8) \\
= 7 \times 138
\]

#### Method A:

<table>
<thead>
<tr>
<th>100</th>
<th>30</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>966</td>
<td>266</td>
<td>56</td>
</tr>
<tr>
<td>-700</td>
<td>-210</td>
<td>56</td>
</tr>
<tr>
<td>-56</td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

\[
7 \left\lfloor \frac{966}{7} \right. \\
= 138 \\
- 700 \\
- 210 \\
- 56 \\
0
\]

#### Method B:

**Conceptual language for this method (all numbers below 966 are in square units):**

- Find the unknown length of the rectangle;
- first find the hundreds, then the tens, then the ones.

- The length gets 1 hundred (units); 2 hundreds (square units) remain.
- 2 hundreds + 6 tens = 26 tens (square units).
- The length gets 3 tens (units); 5 tens (square units) remain.
- 5 tens + 6 ones = 56 ones (square units).
- The length gets 8 ones; 0 remains.

*The “bringing down” steps represent unbundling a remaining amount and combining it with the amount at the next lower place.*

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[https://www.mathedleadership.org/docs/resources/journals/NCSMJournal_ST_Algorithms_Fuson_Beckmann.pdf](https://www.mathedleadership.org/docs/resources/journals/NCSMJournal_ST_Algorithms_Fuson_Beckmann.pdf)

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