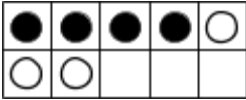
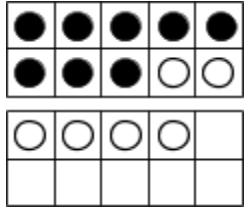
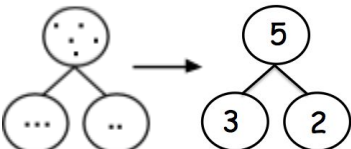
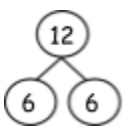
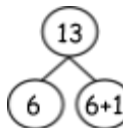
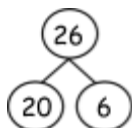
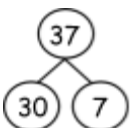
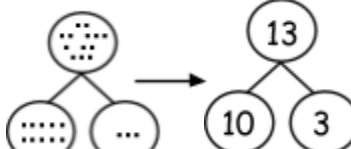
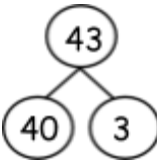
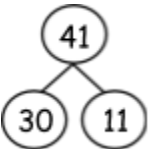
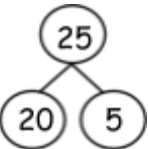
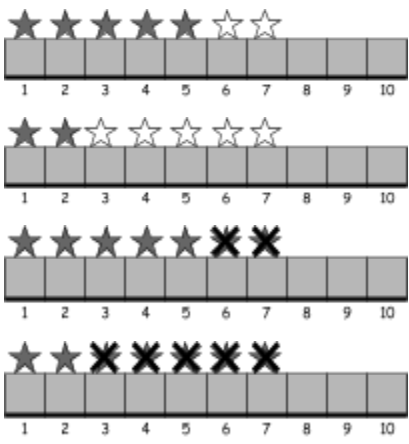
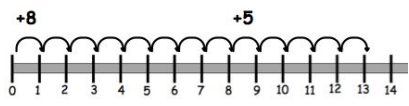
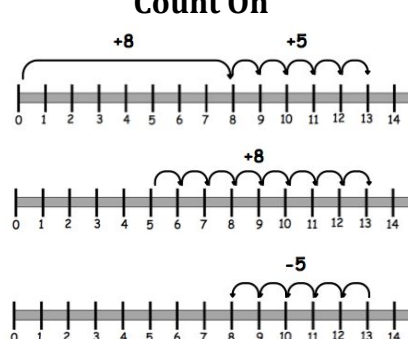
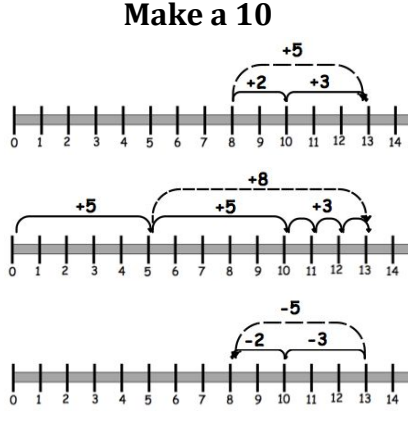
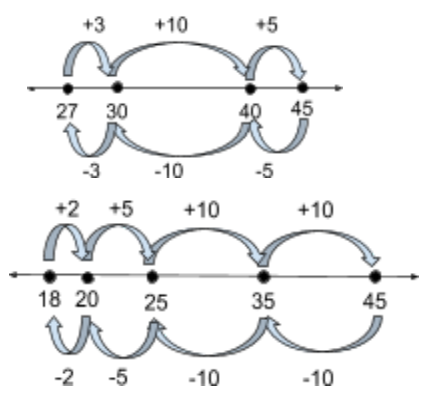


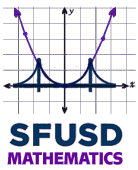
Visual Models: Addition and Subtraction

Kindergarten – Grade 2

	Kindergarten	Grade 1	Grade 2
Ten Frame	 <p>4 and 3 make 7 7 decomposes into 4 and 3 $4 + 3 = 7$ $7 - 3 = 4$ $3 + 4 = 7$ $7 - 4 = 3$</p>		
	 <p>8 and 6 make 14 A 10 and 4 more make 14 14 decomposes into 8 and 6 14 decomposes into 10 and 4 $8 + 6 = 14$ $14 - 6 = 8$ $6 + 8 = 14$ $14 - 8 = 6$</p>		
Number Bond	 <p>3 and 2 make 5 5 decomposes into 3 and 2 $3 + 2 = 5$ $5 - 2 = 3$ $2 + 3 = 5$ $5 - 3 = 2$</p>	<p>Doubles Doubles + 1</p>   <p>$6 + 6 = 12$ $6 + 7 = 13$ $6 + 6 + 1 = 13$ $12 - 6 = 6$ $13 - 6 = 7$ $13 - 7 = 6$</p>	<p>Decompose by Place Value</p> <p style="text-align: center;">$26 + 37$</p>   <p style="text-align: center;">$20 + 30 = 50$ $6 + 7 = 13$ $50 + 13 = 63$</p>
	<p style="text-align: center;">Place Value</p>  <p>10 and 3 make 13 13 decomposes into 10 and 3 $10 + 3 = 13$ $13 - 10 = 3$ $3 + 10 = 13$ $13 - 3 = 10$</p>	<p style="text-align: center;">Place Value</p>  <p>40 and 3 make 43 43 decomposes into 40 and 3 $40 + 3 = 43$ $43 - 40 = 3$ $3 + 40 = 43$ $43 - 3 = 40$</p>	<p style="text-align: center;">$41 - 25$</p>   <p style="text-align: center;">$11 - 5 = 6$ $30 - 20 = 10$ $6 + 10 = 16$</p>

Visual Models: Addition and Subtraction Kindergarten – Grade 2

	Kindergarten	Grade 1	Grade 2																																						
Number Line	 <p style="text-align: center;"> $5 + 2 = 7$ $7 - 2 = 5$ $2 + 5 = 7$ $7 - 5 = 2$ </p>	<p style="text-align: center;">Count All</p>  <p style="text-align: center;">Count On</p>  <p style="text-align: center;">Make a 10</p>  <p style="text-align: center;"> $8 + 5 = 13$ $13 - 8 = 5$ $5 + 8 = 13$ $13 - 5 = 8$ </p>	<p style="text-align: center;">Add/Subtract in Chunks (Open Number Line)</p>  <p style="text-align: center;"> $27 + 18 = 45$ $45 - 27 = 18$ $18 + 27 = 45$ $45 - 18 = 27$ </p>																																						
Tape Diagram		<table style="width: 100%; text-align: center;"> <tr> <td style="width: 33%;">17</td> <td style="width: 33%;">17</td> <td style="width: 33%;">?</td> </tr> <tr> <td><table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td style="width: 20px; height: 20px; text-align: center;">5</td><td style="width: 20px; height: 20px; text-align: center;">?</td></tr></table></td> <td><table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td style="width: 20px; height: 20px; text-align: center;">?</td><td style="width: 20px; height: 20px; text-align: center;">12</td></tr></table></td> <td><table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td style="width: 20px; height: 20px; text-align: center;">5</td><td style="width: 20px; height: 20px; text-align: center;">12</td></tr></table></td> </tr> <tr> <td>$5 + \underline{\quad} = 17$</td> <td>$\underline{\quad} + 12 = 17$</td> <td>$5 + 12 = \underline{\quad}$</td> </tr> <tr> <td>$17 - 5 = \underline{\quad}$</td> <td>$17 - \underline{\quad} = 12$</td> <td>$\underline{\quad} - 5 = 12$</td> </tr> </table> <p style="text-align: center;">Comparison</p> <table style="width: 100%; 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Visual Models: Addition and Subtraction

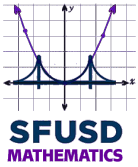
Kindergarten – Grade 2

	Kindergarten	Grade 1	Grade 2
Hundreds Chart		 $4 + 13 = 17$ $17 - 4 = 13$ $13 + 4 = 17$ $17 - 13 = 4$	 $21 + 23 = 44$ $23 + 21 = 44$ $44 - 21 = 23$ $44 - 23 = 21$
Base Ten Blocks		 $34 + 20 = 54$ $54 - 20 = 34$ $20 + 34 = 54$ $54 - 34 = 20$	 $26 + 27 = 53$ $53 - 27 = 26$ $27 + 26 = 53$ $53 - 26 = 27$
Base Ten Blocks		 $34 + 8 = 42$ $42 - 8 = 34$ $8 + 34 = 42$ $42 - 34 = 8$	 $187 + 36 = 223$ $223 - 187 = 36$ $36 + 187 = 223$ $223 - 36 = 187$

Equations in *italics* are part of the “fact family” for the model shown, so students may be able solve them using this information. However based on the CCSS-M, they are beyond the indicated grade level expectations.

This chart shows some examples of how visual models may be used, and is not an exhaustive list.

Updated November 30, 2016



Visual Models: Addition and Subtraction

Kindergarten – Grade 2

Connection to Algorithms: Addition (Grade 3-4)

Place value drawing for all methods



General methods for 2- and 3-digit numbers

D. $\begin{array}{r} 456 \\ + 167 \\ \hline 500 \\ 110 \\ 13 \\ \hline 623 \end{array}$	E. $\begin{array}{r} 456 \\ + 167 \\ \hline 623 \end{array}$	F. $\begin{array}{r} \overset{5}{\cancel{4}}\overset{6}{\cancel{5}}6 \\ + 167 \\ \hline 623 \end{array}$	G. $\begin{array}{r} \overset{1}{\cancel{4}}\overset{1}{\cancel{5}}6 \\ + 167 \\ \hline 623 \end{array}$
--	---	---	---

Methods E, F, and G generalized to 6-digit numbers

E. $\begin{array}{r} 456,789 \\ + 167,189 \\ \hline 623,978 \end{array}$	F. $\begin{array}{r} \overset{5}{\cancel{4}}\overset{6}{\cancel{5}}\overset{8}{\cancel{7}}\overset{9}{\cancel{6}}9 \\ + 167,189 \\ \hline 623,978 \end{array}$	G. $\begin{array}{r} \overset{1}{\cancel{4}}\overset{1}{\cancel{5}}\overset{1}{\cancel{6}}\overset{1}{\cancel{7}}89 \\ + 167,189 \\ \hline 623,978 \end{array}$
---	---	--

Note: Methods E, F and G are all variations in the standard algorithm, but Method E is conceptually clearer and easier.


Fuson, Karen C. and Beckmann, Sybilla. *Standard Algorithms in the Common Core State Standards*. NCSM Journal. Fall/Winter 2012-2013. https://www.mathedleadership.org/docs/resources/journals/NCSMJJournal_ST_Algorithms_Fuson_Beckmann.pdf

Visual Models: Addition and Subtraction Kindergarten – Grade 2

Connection to Algorithms: Subtraction (Grade 3-4)

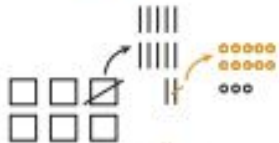
Method A. Ungroup where needed first, then subtract

1. Ungroup hundreds



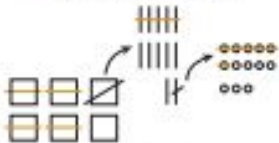
$$\begin{array}{r} 5 \ 12 \\ 623 \\ - 456 \\ \hline \end{array}$$

2. Ungroup tens



$$\begin{array}{r} 5 \ 12 \ 13 \\ 623 \\ - 456 \\ \hline \end{array}$$


3. Subtract everywhere (in either direction)



$$\begin{array}{r} 5 \ 11 \ 13 \\ 623 \\ - 456 \\ \hline 167 \end{array}$$

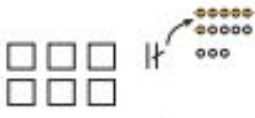
Method B. Alternate ungrouping and subtracting for each column

1. Ungroup tens



$$\begin{array}{r} 1 \ 13 \\ 623 \\ - 456 \\ \hline \end{array}$$


2. Subtract ones



$$\begin{array}{r} 1 \ 13 \\ 623 \\ - 456 \\ \hline 47 \end{array}$$

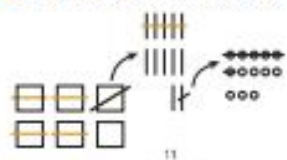
(Error) →

3. Ungroup hundreds



$$\begin{array}{r} 11 \\ 5 \ 7 \ 13 \\ 623 \\ - 456 \\ \hline 7 \end{array}$$

4. Subtract tens, then hundreds



$$\begin{array}{r} 11 \\ 5 \ 7 \ 13 \\ 623 \\ - 456 \\ \hline 167 \end{array}$$

Methods generalized to 6-digit numbers

Method A

Left to right

$$\begin{array}{r} 11 \ 15 \\ 5 \ 12 \ 13 \ 18 \ 19 \ 18 \\ 623,978 \\ - 456,789 \\ \hline 167,189 \end{array}$$

Method B

Right to left

$$\begin{array}{r} 11 \ 15 \\ 5 \ 7 \ 13 \ 18 \ 19 \ 18 \\ 623,978 \\ - 456,789 \\ \hline 167,189 \end{array}$$

Method B

Errors are in red

$$\begin{array}{r} 6 \ 18 \\ 1 \ 13 \ 18 \ 18 \ 18 \\ 623,978 \\ - 456,789 \\ \hline 29 \ 247,189 \end{array}$$

Fuson, Karen C. and Beckmann, Sybilla. *Standard Algorithms in the Common Core State Standards*. NCSM Journal. Fall/Winter 2012-2013. https://www.mathedleadership.org/docs/resources/journals/NCSMJJournal_ST_Algorithms_Fuson_Beckmann.pdf