Math Teaching Toolkit

SFUSD Mathematics Core Curriculum Development Project
Acknowledgements

The SFUSD Math Teaching Toolkit was created in collaboration across departments within the district as well as educational partners from outside.

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Introduction

Jim Ryan, Executive Director of STEM, SFUSD

“...teachers are able to sustain change when there are mechanisms in place at multiple levels of the system to support their efforts. This includes the presence of a supportive professional community of colleagues in the school that reinforces changes and provides continuing opportunities to learn.”
Cynthia Coburn

San Francisco Unified School District is committed to a district-wide implementation of the Common Core State Standards for Mathematics that positively impacts math learning for all students and is both deep and sustainable. This instructional toolkit along with the associated Math Core Curriculum and Professional Development are elements of a system intended to support all teachers in the implementation of the content and pedagogical shifts brought about by the CCSS-M. The three pedagogical strategies included in this toolkit should not be perceived as either exhaustive or limiting. Rather, these are three instructional approaches that lend themselves to the structure and intentionality of the new curriculum and embedded tasks.

We understand that for deep and sustainable change in mathematics, teachers, students, and leaders must grapple with what the rich mathematics of the CCSS-M looks like in the classroom, in student work, in curriculum, and assessments. It is our goal that teachers and site leaders work collaboratively towards a shared vision of good math instruction that develops mathematically proficient students as defined by the CCSS-M. It is our hope that this toolkit provides a common instructional foundation for this collaboration.

Lizzy Hull Barnes, Program Administrator of Mathematics, SFUSD

We all know people who profess to hate math and see it as disconnected from their everyday lives. We often hear statements like “I was never good at math” or “I’m never going to need this.” The SFUSD Mathematics Core Curriculum has been developed by over 300 SFUSD classroom teachers based on the principle that all students, regardless of who they are, can build problem-solving habits of mind through the discipline of mathematics. The Common Core State Standards for Mathematics (CCSS-M) ask teachers and students to grapple with and make sense of math, listen to each other, and defend their reasoning. In fact, this emphasis on discourse transcends mathematics and is also found in the CCSS-ELA, the Next Generation Science Standards, and the CA English Language Development Standards.

Many of us have taught math as a dynamic and creative endeavor for years, and welcome back this spirit of inquiry and collaboration as we would an old friend. For others, this represents a refreshing shift from the way we have been experiencing and teaching mathematics. No matter who you are as a teacher, this toolkit is designed with you and your students in mind. So while the SFUSD Math Core Curriculum itself—the units built upon formative assessment and rich math tasks—is the “what,” this toolkit represents the “how.”

Thank you to each one of the 2,000 math teachers in San Francisco, PreK-12, who will be working to bring rich mathematics to every one of our students every day.
What is a Math Teaching Toolkit?

Mathematics is often seen as a discipline with "the right answer." Many teachers, though, are uncomfortable with this emphasis because it interferes with our efforts to help students express their mathematical thinking, learn from mistakes, experiment effectively, and pursue their mathematical interests. How can we transform the student’s question “Am I right?” into “How can I develop the confidence and judgment that I am on the right track when working on this problem?” In other words, how can we help students keep the locus of sense-making, understanding, and reasoning within themselves as they become confident and skillful problem solvers? At the same time, challenging questions arise for us when we learn to use new materials and improve our teaching strategies. Such questions include:

“How do I know what students are learning?”

“How can I determine if the methods and activities I am using result in improved student learning?”

“How can I balance the development of inquiry skills while covering content and meeting the demand for improved test scores?”

The SFUSD Mathematics Department Math Teaching Toolkit is designed to support teachers and students as we shift from a more directive style of teaching mathematics towards a more inquiry-based style. The umbrella theme of the Math Teaching Toolkit is classroom discourse, which we understand to mean the language that teachers and students use to communicate with each other in the classroom. This theme reflects shifts of pedagogy required to promote the Common Core Standards for Mathematical Practice (see Appendix).

Discourse is the medium for shared sense-making within a community of learners. This is reflected in the Common Core State Standard for Mathematical Practice #1 “Make sense of problems and persevere in solving them.” In addition, discourse makes our thinking public and allows us to negotiate what we mean with others. This is reflected in the Standard for Mathematical Practice #3 “Construct viable arguments and critique the reasoning of others.” This standard states that “students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.”

Classroom discourse offers the opportunity to connect a student’s own everyday language with the specialized language of mathematics, thus reflecting the CCSS-M SMP #6 “Attend to precision.” Student discourse is our window to understand the mathematical strengths and challenges of the whole class as well as an individual student. It presents us a view of what our students know, misconceptions they may have, and how these might have developed. And yet, it is not simply a one-way street. In the process of trying to understand our students’ thinking, we gain perspective into our own mathematical understanding, thereby laying the foundation for a supportive learning community for teachers as well as students.

The SFUSD Math Core Curriculum is intended to promote discourse in the teaching and learning of mathematics. Each unit of study within the Core Curriculum has four rich math tasks as well as lesson series that are premised on group work and positive student-to-student interactions. Our role as teachers is described in broad terms as a facilitator who is listening carefully to students, framing appropriate questions, and mediating competing perspectives. Students are expected to develop problem-solving skills such as: defining problems, formulating conjectures, and discussing the validity of solutions.
The SFUSD Mathematics Department believes that effective teaching does not mean that we simply “stay out of the way.” In fact, we believe that a teacher’s role is proactive and includes:

- Designing lessons that promote student learning through discourse;
- Beginning the lesson with a carefully presented launch;
- Guiding the lesson with facilitated group work and class discussion;
- Ending the lesson intentionally with a closure that brings out students’ insights as well as misconceptions and gives direction for future learning.

A rich math task itself does not necessarily bring out all these desired outcomes. We may give our students a rich math task that is ripe with engaging ideas, only to find that the lesson backfires; we end up with less conversation instead of more, or conversation that does not develop in mathematically significant directions. What went wrong? Do our students have a different understanding than we do about what it means to work well in groups? Did we withhold direct instruction at the wrong times? Do they understand the language of the task?

These are some questions that the SFUSD Math Teaching Toolkit addresses. This toolkit is divided into three sections: Teaching With Rich Math Tasks, Promoting Classroom Discourse, and Collaborative Group Work. Each section highlights one of the three signature strategies (Three Read Protocol, Math Talks, and Participation Quiz/Group Feedback) that we believe will support student learning. In addition, the toolkit contains suggestions for creating classroom norms, orchestrating classroom discussions, setting up collaborative groups, planning for a math day, using notebooks and other tools. Finally, we’ve included the CCSS-M Standards for Mathematical Practice to serve as a reminder that, while there are shifts in content standards, perhaps the most dramatic shift of all is in how we think about teaching and learning mathematics.
Math Tasks

SFUSD units are designed around four tasks. These tasks offer all students opportunities to engage in meaningful and rigorous mathematics that allow for the development of the Standards for Mathematical Practice. They give information about how students are learning the core concepts and skills of the unit.

All tasks are used for formative assessment—gathering information about what students know and are able to do—but they are not tests. The Entry, Apprentice, and Expert Tasks allow for student collaboration and individual accountability without being graded based on an expectation of mastery. The Milestone Task can be used as an individual assessment for grading students (see discussion of rubrics later in this toolkit).

<table>
<thead>
<tr>
<th>Overarching Principles</th>
<th>Tasks support productive struggle.</th>
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<tr>
<td></td>
<td>Tasks...</td>
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<tr>
<td></td>
<td>● are relevant and engaging.</td>
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<td></td>
<td>● have multiple entry points that allow for initial success.</td>
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<td>● have high cognitive demand.</td>
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<td>● allow for divergent ways of thinking.</td>
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<td>● are not scaffolded in ways that reduce cognitive demand.</td>
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<td>● are not timed; students should not be rushed.</td>
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<tr>
<th>Tasks build conceptual understanding.</th>
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<td>Tasks...</td>
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<tr>
<td>● allow students to make connections to prior learning.</td>
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<td>● allow students to answer with multiple representations.</td>
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<tr>
<td>● embed multiple Standards for Mathematical Practice.</td>
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<tr>
<td>● can provide a preview into the next level of learning.</td>
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<table>
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<tr>
<th>Tasks allow students to show what they know and are able to do.</th>
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<td>Tasks...</td>
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<tr>
<td>● cover multiple standards that are central to the unit.</td>
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<td>● contain a balance of skills, concepts, and problem solving.</td>
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<td>● generate student work that a teacher can analyze to measure understanding and to inform instruction in the next lesson series.</td>
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<tr>
<td>Task</td>
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<td>---------------</td>
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<tr>
<td><strong>Entry Task</strong></td>
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<tr>
<td><strong>Apprentice Task</strong></td>
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<td><strong>Expert Task</strong></td>
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<td><strong>Milestone Task</strong></td>
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Math Is for Everyone

The SFUSD Math Core Curriculum is built with all students in mind.

Guiding principles for SFUSD Math:

● All students can and should develop a belief that mathematics is sensible, worthwhile, and doable.
● All students are capable of making sense of mathematics in ways that are creative, interactive, and relevant.
● All students can and should engage in rigorous mathematics through rich, challenging tasks.
● Students’ academic success in mathematics must not be predictable on the basis of race, ethnicity, gender, socioeconomic status, language, religion, sexual orientation, cultural affiliation, or special needs.

What does this mean for our English language learners?

The California English Language Development Standards, adopted in November of 2012, describe three modes of communication in Part I: Interacting in Meaningful Ways: Collaborative, Interpretive, and Productive. The SFUSD PreK-12 Math Teaching Toolkit names three signature pedagogies built around these modes of communication, where all students are reasoning mathematically, defending their reasoning, and listening to and critiquing the reasoning of others in small group and whole group structures. This emphasis on discourse builds students’ proficiency in English and also their proficiency in mathematics. Two should not be seen as separate. Along with the CCSS-ELA and the Next Generation Science Standards (NGSS), the CA ELD standards are part of a national standards movement that places academic discourse at the center of student learning.

What is Universal Design for Learning (UDL)?

Universal Design means lessons that are designed from the beginning to be accessible to all kinds of learners. It comes from the world of architecture, in which buildings are designed to serve everyone, so that a ramp would help not only someone in a wheelchair, but also someone with a baby carriage or someone pulling a dolly. Universal Design for Learning brings this idea of meeting the needs of all people into curriculum design; UDL does not go back and “retrofit” for some students but is built from the outset with all students in mind. In the case of the SFUSD Math Core Curriculum, the central role that the rich math task plays is evidence of this design principle. There are many ways to enter into a problem, and many ways of showing your thinking. A rich math task is often described as having a “low floor and high ceiling”—all learners can access it according to their own competency and background knowledge, and learn alongside peers with different strengths. While the research of UDL is most often associated with Special Education, it is a design principle that we hold at the center of our unit structure, with the core belief that rich mathematics is for all students every day.
Teaching with Rich Math Tasks
SFUSD Signature Strategy #1: Three Read Protocol

What is this strategy?

The Three Read Protocol is a strategy that models how to do a close read of a complex math word problem or task. This strategy includes three separate readings of a math scenario with specific goals. The 1st read is for understanding the context. The 2nd read is for understanding the mathematics. The 3rd read is to elicit inquiry questions based on the scenario.

Why would I use this strategy?

The Three Read Protocol is designed to engage students in sense-making of language-rich math problems or tasks. It deepens student understanding by surfacing linguistic as well as mathematical clues. It focuses attention on the importance of understanding problems rather than blindly trying to solve them. It is an alternative to just simplifying the language. It also allows for natural differentiation within a class of diverse learners.

When do I use this strategy?

This strategy can be used for many math tasks that include complex language structures and/or lend themselves to a variety of interpretations. While this is a particularly useful strategy for EL students, all students can benefit from the deeper understanding of word problem structures and open-ended questioning.

How do I use this strategy?

The Three Read Protocol uses the “problem stem” of a word problem. This is essentially the word problem without the question at the end. The purpose of only presenting the problem stem is to have student focus on the contextual and mathematical information before dealing with any question that is involved. It also allows for students to create their own questions for a given scenario, which is an excellent skill to develop both in math and in reading in general. It is important that the teacher choose the problem carefully and anticipates potential linguistic and mathematical roadblocks the students may encounter.

1. **1st Read: Teacher reads the problem stem orally.**
   The teacher may have visuals to accompany the oral read of the problem stem. Students are asked to listen to the “story” with the goal of turning to a partner and retelling it. They do not have to memorize the information. After the Turn-and-Talk, the teacher asks students to volunteer information they remember from the story. Teachers and students ask clarifying questions about the vocabulary as needed.

2. **2nd Read: Class does choral read or partner read of the problem stem.**
   The teacher projects the problem stem so the whole class can see it. The teacher leads the class either in a choral read of the problem or has partners read the problem orally to each other. Choral read is preferable because it allows all students to participate without excessive pressure, but partner reads can work fine if that is a better fit to the classroom culture or age of students. The teacher explains that math stories usually have information about quantities, which are numbers and the units that are being counted. An example is 25 cats, where “25” is the quantity and “cat” is the unit. Sometimes the quantities are implied. For example, “some cats” implies a quantity but we do not know what it is. There can also be implied units. An example is “I have one at home.” The implied unit in this case depends entirely on the context of the story. **Bottom line:** The discussion of quantities and units can be important for focusing student attention, but whether the teacher delves deeply into the explicit and implicit information depends entirely on the math problem and the needs of the students.
3. **3rd Read: Class or partner read the problem stem orally one more time.**
   The teacher asks students to do one more read of the “story” and ask them to think “What is missing to make this a good math problem?” Students volunteer their answers to that question. Responses will likely vary because many students assume there is a question without actually reading one. Without correcting student responses, the teacher probes until the class decides that a question is missing. The teacher asks “Is there only one question that we can ask of this story?” Students responses may vary, but there are usually many different questions that can be asked of almost any scenario. The teacher asks partners to determine at least two questions that can be asked of the problem stem. After a few minutes, a couple of students volunteer their questions. The teacher writes a couple of the questions and clarifies language as appropriate. After each question, the teacher asks the class “Can this question be answered with the information from this story?” and the class discusses why or why not.

4. **Students work in collaborative groups on the problem.**
   The teacher can choose to either have students work in groups on a question they choose or a question another student volunteered, or the teacher can pose his/her own question for the class to work on. If groups are asked to choose their own questions, it is important that the teacher circulate and clarify expectations around the work. Another strategy is to use one of the questions volunteered by a student and have the whole class work on it. Finally, this can be an opportunity to differentiate the math work because the range of possible questions to a problem stem is broad.

**In summary:**

<table>
<thead>
<tr>
<th>What the teacher does</th>
<th>What the students do</th>
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</thead>
<tbody>
<tr>
<td><strong>1st Read</strong></td>
<td></td>
</tr>
<tr>
<td>• Identifies appropriate problem stem</td>
<td>• Sit with a partner</td>
</tr>
<tr>
<td>• Anticipates linguistic and/or mathematical challenges</td>
<td>• Listen to the “story”</td>
</tr>
<tr>
<td>• Creates visuals to support understanding</td>
<td>• Turn to partner to discuss the “story” in his/her own words</td>
</tr>
<tr>
<td>• Orally reads the “story” (problem stem)</td>
<td>• Say what they remember of the story</td>
</tr>
<tr>
<td><strong>2nd Read</strong></td>
<td></td>
</tr>
<tr>
<td>• Has “story” available on overhead, projector, or poster</td>
<td>• Read chorally with the class or with a partner</td>
</tr>
<tr>
<td>• Leads class in choral read or partner read</td>
<td>• Volunteer quantities and units he/she identifies</td>
</tr>
<tr>
<td>• Leads discussion of quantities and units</td>
<td></td>
</tr>
<tr>
<td><strong>3rd Read</strong></td>
<td></td>
</tr>
<tr>
<td>• Has partners read with specific goal in mind “What is missing to make this a math problem?”</td>
<td>• Read one more time with partner</td>
</tr>
<tr>
<td>• Leads discussion of potential questions</td>
<td>• Brainstorm with partner several questions that could be asked of this scenario</td>
</tr>
<tr>
<td>• Clarifies language of the questions, as needed</td>
<td>• Volunteer question for the problem stem</td>
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</table>

**Considerations for use of the Three Read Protocol**

1. **Is the problem stem sufficiently interesting as a story?**
   - It does not need to be long, but it should have a narrative purpose.

2. **Does the problem stem have quantities, both explicit and implicit?**
   - Ideally it has easily identifiable explicit quantities, but implicit ones as well for a richer discussion and potentially more interesting math investigations.

3. **Does the problem stem have extraneous explicit quantities or a variety of implicit quantities?**
   - This strategy can model how to comprehend math problems with this characteristic and teach students how to discern salient information.

4. **Is the language of the problem stem likely to create obstacles for EL students or is the context of the problem likely to be unfamiliar to students from diverse backgrounds?**
   - Surfacing the language structure and contextual clues within the problem stem allows students to focus on the mathematical structures and evaluate the reasonableness of their work.
Sample Problem Stems

Animal Shelter Problem

Tasha wants a pet. She goes to the animal shelter to ask how much it will cost to adopt and care for a dog. The vet at the shelter tells her that big dogs have an adoption fee of $200; a vaccination fee of $300; and they eat about 35 pounds of food per month. Small dogs have an adoption fee of $300; a vaccination fee of $450; and they eat about 18 pounds of food per month. The vet says that dog food costs about $3 per pound.

Judy's Berries

Judy loves to eat berries for breakfast, lunch, and dinner. She sees that Clear Lake School is having a fundraiser to raise money for a new playground. The students are selling fruit baskets to raise the money. Strawberries sell for $3 per basket. Blueberries sell for $4 per basket. Raspberries sell for $5 per basket. Judy has $20 to spend on berries.

Joining a Gym

Carlos wants to join a gym. The gym offers three membership options. The first one is called "Pay as you go" and costs $6 each time you work out. The second one is called "Regular deal" and costs $50 per month and $2 each time you work out. The third one is called "All-in-one price!" and costs $100 per month for unlimited use of the gym.

Squirrels and Their Acorns

Austin likes to watch squirrels find and store acorns for the winter. Brown Squirrels can carry two acorns at a time. Gray Squirrels can carry three acorns at a time, and Black Squirrels can carry five acorns at a time. There is a pile of 24 acorns.

Further Resources

SFUSD Mathematics Department website:  www.sfusdmath.org/3-read-protocol.html
Rule of Four

What is the “Rule of Four”?
The “Rule of Four” is a way to think about math both at the entry point of a task and in the representation of math thinking. Showing our thinking through multiple representations helps us have a stronger and deeper understanding of the mathematics. It also allows us to see connections across concepts and topics in mathematics.

Why use the “Rule of Four”?
When we strive to represent our understand using the “Rule of Four,” we are asking ourselves to find deeper connections both within and across concepts. In addition, it validates multiple perspectives in mathematics.

When to use the “Rule of Four”
The “Rule of Four” is appropriate for most math work. How much it is emphasized depends on the context.

How to use the “Rule of Four”
Student may not be familiar and/or fluent with representing their work in a variety of ways. Explicit modeling from the teacher and giving students many opportunities to practice representing their work in multiple ways is an effective way to teach students to think about their math work in this fashion. It is equally important that students make connections among the representations. These connections lead to a deeper understanding over time.
A “Gallery Walk” is an activity that allows students opportunities to discuss and display their work around a room much like artists would display their artistic pieces in an exhibit. It is a non-threatening way for students to receive feedback on their work, check their understanding, and see multiple solution paths for a given math task. It is an ideal opportunity for students to see and discuss multiple ways of approaching and representing math thinking (as described in the “Rule of Four”).

Here is one way of conducting a Gallery Walk in your classroom.

1. An assigned task/activity is worked on by a team of 3–6 students that form Home Base Teams.

2. When the teams finish their collaborative work poster, they each display their work around the classroom. This can take the form of a poster, or it can be a simple as laying their work out on the tables.

3. Inform the teams that they will visit each one of the posters to generate a discussion about the work/poster that each group did. There are (at least) two options to direct students’ observations and thinking during the Gallery Walk itself.
   a. Students may post comments and/or clarifying questions on their peers’ work with sticky notes. Provide, or have students generate, clear guidelines to help groups write useful and appropriate comments. These might include:
      ● Address the math, not the person.
      ● Disagree respectfully.
      ● Keep this a safe math space.
   b. Students may fill out a Gallery Walk Response Sheet instead of posting their thinking directly on their classmates’ work. In this case, you may choose to direct students to pick one or more posters to focus their thinking on. Questions might include all of those above plus:
      ● What are some mathematical characteristics of the work that fit...? (Insert an aspect of the math content that you are focusing on.)
      ● What do you notice that is similar among all or most of the posters?
      ● What do you notice that is different on all or most of the posters?
4. Once each team has visited each of the posters, they return to their poster and review the comments left by the other groups or discuss their Gallery Walk Response Sheets. If there is time, and after considering the work that they saw on others' posters and/or the comments and questions that were left, they may modify their work on the poster.

5. Collect any clarifying questions that were posted on the posters with sticky notes or on the Response Sheets and begin a whole group discussion.

6. Each clarifying question is directed to the collaborative group that generated and/or created the poster. To facilitate the discussion, you read each clarifying question and then write them on a white board/smart board/chart paper, identifying each question by team name/number (i.e., Group 1, 2 or by the Group's invented name, if appropriate).

7. Then a discussion ensues. You give each team the floor when their question is brought up. The team is given the opportunity to answer the clarifying question, which can be done by one representative from the group or by all the group members chiming in when appropriate and/or when there is a pause after a team member adds a final comment.

As your class develops a comfortable routine around Gallery Walks, you can extend the activity by asking students to generate the questions that they will respond to as they observe the posters, either on sticky notes or on Response Sheets. In some cases, you can have students generate prompts or questions that they would like observers to respond to about their specific poster.
Rubrics

Many of the tasks in the SFUSD Math Core Curriculum have rubrics. Although rubrics can be used to evaluate student work and provide a numerical grade, they are just as important for providing specific feedback on critical elements of the task and the student work. There are many ways to use a rubric. Often it is helpful for students to see how they will be evaluated before they begin a task. Another approach is to let them complete the tasks, then review the rubric, and then make changes before they submit their work.

Many rubrics in the SFUSD Math Core Curriculum are point rubrics, but if you want or need to use the rubric scores as letter grades, you may find yourself in a bind. Often the easiest way to use these scores is to turn them into a percentage. Mathematically this is accomplished by dividing the points earned by the number of points possible. This method alone will not give an accurate picture of student achievement because it may well depend on the number of points within the rubric. For example, if a student scores three points on a four-point rubric, that student has performed well. Three out of four points mathematically, though, is only a score of 75 percent. In many grading systems, the student is now left with an undeserved C.

To be fair to students and to calculate a more accurate reflection of student performance, the numbers can be manipulated slightly to achieve a grade that is more indicative of the quality of the product. Here are two options you may want to choose from:

Option One:

Turn the rubric score into a percent by dividing the points earned by the points possible, and then use the scale provided. Example: Student earns 10 out of 12 points. 10 ÷ 12 = 83%, 83% = B

- 88 – 100 = A
- 75 – 87 = B
- 62 – 74 = C
- 50 – 61 = D
- 0 – 50 = F

This is a suggested point scale that seems to work fairly well. Percents can be adjusted up or down to best meet the needs of the students in your class.

Option Two:

Determine the total points possible for the rubric. Divide the total possible by 5 to determine the increments for each point group. This is done because there are 5 grades in the A, B, C, D, F grading scale. Example: Rubric points possible total = 12, 12 ÷ 5 = 2.4

The (quotient) answer determines the highest point value for the F group. F = 0–2.4 points

To determine the highest value for the D group add the quotient (answer) to the highest point value in the F group. D = 2.41–4.8 points

To determine the highest value for the C group add the quotient (answer) to the highest point value in the D group. C = 4.81–7.2 points

To determine the highest value for the B group add the quotient (answer) to the highest point value in the C group. B = 7.21–9.6 points

To determine the highest value for the A group add the quotient (answer) to the highest point value in the B group. A = 9.61–12
### Sample Point Rubric (Based on MARS Task Rubrics, SVMI)

**Designing a Box Rubric**

The core standards addressed by this task are:

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

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<th>Rubric</th>
<th>points</th>
<th>section points</th>
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<tbody>
<tr>
<td>1</td>
<td>Gives correct answer: <strong>2,880 cm³</strong></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Shows how they figured it out such as:</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>Area of base: $\frac{1}{2}(15 + 5) \cdot 2 = 120 \text{ cm}^2$ (or calculates base using an alternate method)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Volume = $120 \cdot 24 = 2,880 \text{ cm}^3$</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>Gives correct answer: <strong>16 cm</strong></td>
<td>1</td>
<td></td>
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<tr>
<td></td>
<td>OR gives a correct answer based on an incorrect response for #1</td>
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<td></td>
<td>Shows how they figured it out such as:</td>
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<tr>
<td></td>
<td>$9 \cdot 20 \cdot h = 2880$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$180 \cdot h = 2880$</td>
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<tr>
<td></td>
<td>$2880 ÷ 180 = 16$</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>Gives dimensions for a box that has a volume of 2,880 cm³</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>OR for a box with the volume they got for #1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>For example, 10 cm x 4 cm x 72 cm = 2880 cm³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Correctly calculates the surface area of Box A, <strong>1,334 cm²</strong>, showing correct work such as:</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Front and back: $2(120) = 240$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Left and right: $2(24 \cdot 13) = 624$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Top: $5 \cdot 24 = 120$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bottom: $15 \cdot 24 = 360$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$240 + 624 + 120 + 360 = 1344$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly calculates the surface area of Box B, <strong>1,288 cm²</strong>, Showing correct work such as:</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>$2(9 \cdot 16) = 288$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$2(20 \cdot 16) = 640$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$2(20 \cdot 9) = 360$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$288 + 640 + 360 = 1288$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Indicates that Box A uses more cardboard</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Total Points** 8
Sample 3-Point Holistic Rubric (From *Everyday Mathematics*)

<table>
<thead>
<tr>
<th>Points</th>
<th>Description</th>
</tr>
</thead>
</table>
| 3 Points | A three-point response is complete and correct. This response:  
- demonstrates a thorough understanding of the mathematical concepts and/or procedures embodied in the task.  
- indicates that the student has completed the task correctly, using mathematically sound procedures.  
- contains clear, complete explanations and/or adequate work when required. |
| 2 Points | A two-point response is partially correct. This response:  
- demonstrates partial understanding of the mathematical concepts and/or procedures embodied in the task.  
- addresses most aspects of the task, using mathematically sound procedures.  
- may contain an incorrect solution but applies a mathematically appropriate process with valid reasoning and/or explanation.  
- may contain a correct solution but provides incomplete procedures, reasoning, and/or explanations.  
- may reflect some misunderstanding of the underlying mathematical concepts and/or procedures. |
| 1 Point | A one-point response is incomplete and exhibits many flaws but is not completely incorrect. This response:  
- demonstrates only a limited understanding of the mathematical concepts and/or procedures embodied in the task.  
- may address some elements of the task correctly but reaches an inadequate solution and/or provides reasoning that is faulty or incomplete.  
- exhibits multiple flaws related to a misunderstanding of important aspects of the task, misuse of mathematical procedures, or faulty mathematical reasoning.  
- reflects a lack of essential understanding of the underlying mathematical concepts.  
- may contain a correct numerical answer but required work is not provided. |
| 0 Points | A zero-point response is completely incorrect, irrelevant, or incoherent, or a correct response that was arrived at using an obviously incorrect procedure. |
Chess Club

There were 24 boys and 20 girls in a chess club last year. This year the number of boys increased by 25% but the number of girls decreased by 10%.

1. Was there an increase or decrease in overall membership?

<table>
<thead>
<tr>
<th>0 points</th>
<th>1 point</th>
<th>2 points</th>
</tr>
</thead>
<tbody>
<tr>
<td>The response is incorrect or there is no response.</td>
<td>The answer is correct, but the work is incorrect OR the answer is incorrect due to a computation error made using the correct operations.</td>
<td>The response is correct and complete. Sample correct answer: There is an increase in overall membership. 24 boys + 20 girls = 44 members 25% of 24 = 6, and 24 + 6 = 30 boys 10% of 20 = 2, and 20 – 2 = 18 girls 30 boys + 18 girls = 48 members</td>
</tr>
</tbody>
</table>

2. Find the overall percent change in membership of the club.

<table>
<thead>
<tr>
<th>0 points</th>
<th>1 point</th>
<th>2 points</th>
</tr>
</thead>
<tbody>
<tr>
<td>The response is incorrect or there is no response.</td>
<td>The answer is correct, but the work is incorrect OR the answer is incorrect due to a computation error made using the correct operations.</td>
<td>The response is correct and complete. Sample correct answers: 9% or 9.1% 44 + 4 = 48 4 is x% of 44 4 =  ( \frac{x}{100} ) ( \times ) 44 400 = 44x 400 ( \div ) 44 = x x = 9.09</td>
</tr>
</tbody>
</table>

Sample Constructed Response Rubric (from SFUSD Math Core Curriculum Unit 7.2)
Promoting Classroom Discourse
SFUSD Signature Strategy #2: Math Talks

What is this strategy?

Math Talks are teacher-led, student-centered techniques for building math thinking and academic discourse. They are intended to last for 10–15 minutes. Math Talks can be centered on any math topic. However, they are not used to introduce math content. Rather, Math Talks are best when the content is generally familiar to students up to their Zone of Proximal Development.

Why would I use this strategy?

Math Talks serve to further understanding of math content while addressing Math Practice Standard #3: *Construct viable arguments and critique the reasoning of others.* They give students the opportunity to develop flexibility and fluency with mental visualization and computation. They offer opportunities to revisit math topics as well as deepen understanding by sharing multiple ways of thinking about a concept or skill.

When do I use this strategy?

This strategy can be used at any time, but is often done at the beginning of a math class. Because it does not need to be focused on the lesson’s content, the content of the Math Talk can vary according to the needs of the students. Math Talks generally happen 2 or 3 times a week for 10–15 minutes each.

How do I use this strategy?

Teachers deliberately set up a safe environment where each child’s thinking is valued.
Students practice making their thinking explicit.
Everyone practices understanding each other’s thinking.

1. **Teacher presents the problem.**
   A problem is presented to the whole class or a small group. Computation problems are always presented horizontally (e.g., $43 + 35 = ?$), so as to encourage mental strategies rather than reliance on algorithms.

2. **Students think about the problem.**
   Students are given time (1–2 minutes) to silently, mentally think about the problem and try to find an answer. They signal quietly to the teacher (e.g., with a thumb up against their chest) when they have an answer.

3. **Students share their answers.**
   A few students volunteer to share their answers and the teacher records them on the board. Without judgment, the teacher records answers where all students can see. The teacher continues to take answers until all students’ answers have been shared. Teacher can also ask the students to Turn-and-Talk with a partner before sharing answers.

4. **Students share their thinking.**
   Students share how they got their answers with a partner or with the larger group. Any student can provide an explanation to any answer on the board. Equity sticks can be used to ensure every student has an equal opportunity to share. The teacher records the students’ name and thinking. The teacher asks questions that help students express themselves, understand each other, and clarify their thinking to make sense of the problem and its solution(s). Multiple ways of solving problems are emphasized. The student’s name is attached to the solution.
### Sample Math Talks By Level

<table>
<thead>
<tr>
<th></th>
<th>K-2</th>
<th>3-5</th>
<th>6-8</th>
<th>High School</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dot Talks</strong></td>
<td>![ DOT TALKS IMAGE ]</td>
<td>![ DOT TALKS IMAGE ]</td>
<td>![ DOT TALKS IMAGE ]</td>
<td>Which is greater 86 x 38 or 88 x 36?</td>
</tr>
<tr>
<td>How many dots do you see?</td>
<td>![ DOT TALKS IMAGE ]</td>
<td>![ DOT TALKS IMAGE ]</td>
<td>![ DOT TALKS IMAGE ]</td>
<td></td>
</tr>
<tr>
<td><strong>How many triangles do you see?</strong></td>
<td>![ TRIANGLES IMAGE ]</td>
<td>15 x 18</td>
<td>![ TRIANGLES IMAGE ]</td>
<td>How many tiles in figure 10?</td>
</tr>
<tr>
<td><strong>Number Strings</strong></td>
<td>12 + 12</td>
<td>15 x 18</td>
<td>![ NUMBER STRINGS IMAGE ]</td>
<td></td>
</tr>
<tr>
<td>12 + 13</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13 + 13</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13 + 14</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Place ½ and 2 ½ on this number line</strong></td>
<td>![ NUMBER LINE IMAGE ]</td>
<td>Estimate what 32% of 647 is.</td>
<td>![ NUMBER LINE IMAGE ]</td>
<td>![ NUMBER LINE IMAGE ]</td>
</tr>
<tr>
<td><strong>Always, Sometimes, Never:</strong></td>
<td>![ ALWAY SOME NEVER IMAGE ]</td>
<td>![ ALWAYS SOME NEVER IMAGE ]</td>
<td>![ ALWAYS SOME NEVER IMAGE ]</td>
<td></td>
</tr>
<tr>
<td>If you double the numerator of a fraction, you double the size of the fraction.</td>
<td>![ ALWAY SOME NEVER IMAGE ]</td>
<td>![ ALWAYS SOME NEVER IMAGE ]</td>
<td>![ ALWAYS SOME NEVER IMAGE ]</td>
<td></td>
</tr>
</tbody>
</table>
| \[
\frac{a}{b} \rightarrow \frac{2a}{b}
\] | ![ ALWAY SOME NEVER IMAGE ]    | ![ ALWAYS SOME NEVER IMAGE ]    | ![ ALWAYS SOME NEVER IMAGE ]   |                                   |
| **How many triangles do you see?** | ![ TRIANGLES IMAGE ]           | ![ TRIANGLES IMAGE ]           | ![ TRIANGLES IMAGE ]           |                                   |

### Math Talk Layers

As you begin to implement Math Talks in your classroom, you will want to keep them simple. Your goal might be to have 2 or 3 students share their thinking, which you capture and record without much comment or questioning.

**Initial Implementation:**

- Provide a safe environment.
- Start with easier problems so that students can learn the routine.
- Present problems horizontally.
- Provide quiet think time and a silent signal.
- Capture student thinking as faithfully as you can.
- Accept, respect, and consider all answers.
- Develop your poker face. Respond neutrally to students’ comments.

As you and your students’ familiarity with Math Talks grows, you will find that you can begin to ask questions and probe their thinking.
Here are some clarifying and probing questions to try:
❖ Where did you get this number?
❖ How did you get this?
❖ Why did you do this operation?
❖ Do you mean this?
❖ Is this how you thought of it?
❖ So you are saying that…?

These questions help students in breaking down their thinking and explaining the steps they went through.

**Share the Why with Students**

Give the students the rationale behind the Math Talk. Let them know that they have such great thinking going on that we can't see and this gives them a chance to share what's going on in their brains. It also gives everyone a chance to learn from each other and informs the teacher about what they know and what we might need to work on.

**Adding Layers**

❖ Ask students if they thought of the problem in the same or a different way. (This can be done verbally or with a signal, e.g., pat your head if you thought of it the same way.)
❖ Have students Turn-and-Talk. Use this strategy when many students want to talk and may not have a turn individually; when you want to generate more answers and/or discussion from students; or when students need time and practice articulating their math ideas and strategies before sharing with the whole group.
❖ Begin to ask questions that connect students thinking to each other:
   ➢ Who has a question for ____?
   ➢ Who can paraphrase what ____ is saying?
   ➢ Who can explain what ____ is thinking?
   ➢ Do you agree or disagree with what they said? Can you explain why?
❖ Point out similarities and differences between different strategies.
❖ Ask students to point out similarities and differences between different strategies.

**Responding to Math Talks**

As you become increasingly comfortable with using Math Talks, you will find yourself adjusting them and incorporating them into your pedagogical repertoire.
❖ Design new Math Talks based on issues that arise during math instruction.
❖ Design math instruction based on confusions that arise during Math Talks.
❖ Create Class Strategy posters that summarize different strategies that your class is using in Math Talks.
❖ Simplifying Math Talks when students have difficulty. Using smaller numbers can help students access a strategy that they can then apply to larger numbers.
❖ You can offer more than one problem during a Math Talk and allow students to choose the one they want to solve. For example, 13 X 12 and 15 X 17 both get at multi-digit multiplication, but one uses numbers that may be easier for students to keep in their heads as they solve the problem mentally.

**Further Resources**

SFUSD Mathematics Department website: [www.sfusdmath.org/math-talks.html](http://www.sfusdmath.org/math-talks.html)
San Diego Unified School District website: [www.sandi.net/Page/33501](http://www.sandi.net/Page/33501)
*Number Talks - Helping Children Build Mental Math and Computation Strategies* (K-5) Sherry Parrish
Facilitating productive discussions about mathematics is very challenging for any teacher. Some lessons can end effectively with a “share and summarize.” At other times, though, a more purposeful discussion is needed to bring out the key mathematics of a lesson.

A key component of productive discussion is teacher facilitation. This facilitation is not accidental and cannot, generally, happen on the fly.

Here are 5 concrete steps that can help improve the quality of mathematics discussion in your class.

1. **Anticipating** likely student responses to mathematical tasks.

   Involves envisioning potential student responses, strategies (correct or incorrect), representations, procedures, and interpretations.

2. **Monitoring** students’ actual responses to the tasks.

   Involves paying close attention to students’ mathematical thinking as they work on a problem. Commonly done by circulating around the classroom during group work.

3. **Selecting** student response to feature during the discussions.

   Involves choosing particular students to present their work because of the mathematical responses. These responses need not be chosen solely because they are correct, but rather because they emphasize different approaches to the problem. In fact, it may be advantageous to choose incorrect responses to highlight how and why they are incorrect. This choice can highlight a variety of responses or strategies for a task, or it can show a progression from simple to complex representation. Make sure over time that all students feel they are authors of mathematical ideas.

4. **Sequencing** student responses during the discussions.

   Involves purposeful ordering of the featured student responses in order to make the mathematics accessible to all students. This also helps build a mathematically coherent story line during whole class discussion.

5. **Connecting** student responses during the discussions.

   Involves encouraging students to make mathematical connections between different student responses. This helps ensure that key mathematical ideas remain the focus of the lesson debrief.

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1Based on NCTM publication *5 Practices for Orchestrating Productive Mathematics Discussions* by Margaret Smith and Mary Kay Stein

SFUSD Mathematics Department, June 2014, sfusdmath.org
Effective Questioning

You can promote discourse and stimulate student thinking through effective questioning. This, in turn, develops the habits of mind suggested by the Standards for Mathematical Practice. Here is a list of questions from the *Professional Standards in Teaching Mathematics*, grouped into categories that reflect the mathematical practices.

❖ **Helping students work together to make sense of mathematics:**
  ➢ “What do others think about what Janine said?”
  ➢ “Do you agree? Disagree?”
  ➢ “Does anyone have the same answer but a different way to explain it?”
  ➢ “Do you understand what they are saying?”

❖ **Helping students to rely more on themselves to determine whether something is mathematically correct:**
  ➢ “Why do you think that?”
  ➢ “Why is that true?”
  ➢ “How did you reach that conclusion?”
  ➢ “Can you make a model to show that?”

❖ **Helping student learn to reason mathematically**
  ➢ “Does that always work?”
  ➢ “Can you think of a counterexample?”
  ➢ “How can you prove that?”
  ➢ “What assumptions are you making?”

❖ **Helping students learn to conjecture, invent, and solve problems:**
  ➢ “What would happen if…? What if not?”
  ➢ “Do you see a pattern?”
  ➢ “What is alike and what is different about your method and her method to solve the problem?”
  ➢ “Can you predict the next one? What about the last one?”

❖ **Helping students to connect mathematics, its ideas, and its applications:**
  ➢ “How is this process like others that you have used?”
  ➢ “How does this relate to _______?”
  ➢ “Have you ever solved a problem like this before?”
  ➢ “Can you give me an example of _______?”

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2 Adapted from *NCSM Great Tasks for Mathematics, 6-12* by Schrock, Norris, Pugalee, Seitz, and Hollingshead, 2013
SFUSD Mathematics Department, June 2014, sfusdmath.org
Collaborative Group Work
SFUSD Signature Strategy #3: Participation Quiz/Group Feedback

What is this strategy?

A Participation Quiz/Group Feedback is a strategy to help establish or reinforce norms for group work in a cooperative environment. While students work together in their group on a math task, the teacher takes public notes—on a document camera, white board, chart paper, or overhead projector—about the quality of their group work (social moves) and the quality of their mathematical discussions (math moves). The teacher can take notes on how students work together, their use of classroom norms, or the specific language they use to communicate their mathematical ideas.

Why would I use this strategy?

Publically taking notes on students' interactions allows the teacher to communicate the behaviors they wish to encourage and value, as well as mitigating perceived status differences between students—that is, highlighting strengths of students who are not perceived to be strong in math. Some teachers assign each group a grade at the end of a Participation Quiz/Group Feedback. Other teachers prefer to focus on the feedback rather than giving it a score. This protocol might be named differently, for example, “Group Work Feedback,” to reflect the teacher’s objective.

When do I use this strategy?

This strategy can be used whenever students are working in collaborative groups.

How do I use this strategy?

1. Choose a worthy task.
   The teacher chooses a task that is accessible, challenging, important, and requires students to read and talk together. If a task is too hard, the teacher may spend more time answering group questions than observing, and if a task is too routine students will naturally do these individually since little collaboration will be required.

2. Decide on a focus.
   The teacher decides which group norms or Standards of Mathematical Practice he/she wants students to focus on. This decision depends on the context of the classroom. Early in the year, the teacher may focus on establishing norms, such as getting a quick start (reading problem promptly and making sure group understands), working together (heads leaning in and working in the middle of the group), and asking the group questions before asking the teacher. Later in the year, the teacher may focus on refining a norm that a particular class is struggling with, such as making statements with reasons, or the teacher may choose to highlight strengths of specific students that have low status (students who are not generally seen as strong in math).

3. Communicate the focus to students.
   The teacher lets his/her class know that the lesson will be structured as a Participation Quiz/Group Feedback. The teacher is clear about what he/she is looking for and uses language that students understand. The teacher explains that he/she will publically record a snapshot of the students working together in their groups.

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2 This description is based on Smarter Together—Collaboration and Equity in the Elementary Math Classroom by Featherstone, Crespo, Jilk, et al. and the Instructional Toolkit for Mathematics produced by Oakland Unified School District.
For example, the teacher can say that he/she is looking for:

- “because” statements (addresses “making statements with reasons”).
- students leaning in (addresses “working together on the same problem”).
- group questions only.

As students work, the teacher publically records statements about how groups are working together. This can be done on a document camera, white board, chart paper, or overhead projector. The recording sheet is split into as many spaces as there are groups (see example diagram below). Sometimes groups do not notice this public documentation, while other times they pay attention and change their behaviors to meet the norms.

4. Debrief the notes taken.
The teacher takes time before the end of class or in the middle of the task to debrief. Time is given for students to read comments. The teacher highlights key evidence that supports the group work norms. The focus on group work norms to start the class and then end the lesson can be a powerful way to reinforce the kinds of cooperative behaviors that teachers want to establish.

When students are used to seeing this structure, teachers can use these public notes as a “quiz” to assess students and groups on their group work skills. Generally, the focus should be on positive behaviors, although over time honest critiques of behavior may be included as well.

Example Participation Quiz / Group Feedback Diagram

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>QS (Quick Start)</td>
<td>QS</td>
</tr>
<tr>
<td>Miguel, “Can you re-read the problem?”</td>
<td>So, I think this means…</td>
</tr>
<tr>
<td>I said this was 3x + 2 because…</td>
<td>Heads leaning in</td>
</tr>
<tr>
<td></td>
<td>“What did you mean by…”</td>
</tr>
<tr>
<td></td>
<td>“So, x means… do you get it?”</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Group 3</th>
<th>Group 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>QS</td>
<td>Reading problem</td>
</tr>
<tr>
<td>“I’m not sure what to do. Can you…”</td>
<td>“Melissa, explain to me how you got the equation.”</td>
</tr>
<tr>
<td>Using a table of points</td>
<td></td>
</tr>
<tr>
<td>I think the pattern is +3; see, look at…</td>
<td></td>
</tr>
<tr>
<td>Using “because” statements</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Group 5</th>
<th>Group 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>QS</td>
<td>QS</td>
</tr>
<tr>
<td>“Can you repeat your idea?”</td>
<td>“I don’t understand the where the 2 shows up on the graph.”</td>
</tr>
<tr>
<td>Let’s make a graph.</td>
<td>“Oh…I see the adding 3 in the table and graph; now…that makes more sense.”</td>
</tr>
<tr>
<td>“Something’s not right. What do you think?”</td>
<td>Students leaning in on work in the middle</td>
</tr>
<tr>
<td>Explaining graph and equation.</td>
<td></td>
</tr>
</tbody>
</table>

Further Resources

SFUSD Mathematics Department web site: [www.sfusdmath.org/participation-quiz--group-feedback.html](http://www.sfusdmath.org/participation-quiz--group-feedback.html)

*Smarter Together-Collaboration and Equity in the Elementary Math Classroom* Featherstone, Crespo, Jilk, et. al.
Class Norms

Class norms:
❖ are behaviors that are expected of all members of the classroom community, adults and students.
❖ cover large sets of behaviors.
❖ apply to all situations all the time.
❖ are stated in a positive tone.
❖ are few in number.
❖ are developed by the classroom community.

Norms Versus Rules

A positive classroom climate enhances student interaction. An important step in creating this climate is agreeing on norms for the class. Unlike rules, which are set by the teacher for classroom safety and efficiency, norms should arise from the students’ own feeling about how they want to be treated by others. Norms are agreements among the members of the community about how they will treat one another. Over the year, students explore how they are acting on prosocial values, and how they want to be treated and to treat others this year. The norms help the students to be accountable to the community and to act on the prosocial values of responsibility, respect, fairness, caring, and helpfulness. In an environment in which students live by norms, they have many opportunities to take responsibility for their own behavior.

Both norms and rules are necessary in any classroom. Examples of each are shown below. Rules should be established by the teacher on the first day of school. Phrasing the rules using a positive tone contributes to the sense of community.

<table>
<thead>
<tr>
<th>Rules</th>
<th>Norms</th>
</tr>
</thead>
<tbody>
<tr>
<td>❖ Respect the school and school property.</td>
<td>❖ We will treat each other with respect.</td>
</tr>
<tr>
<td>❖ Respect teachers, staff, and all guest adults.</td>
<td>❖ We will help each other.</td>
</tr>
<tr>
<td>❖ Respect others and be polite.</td>
<td>❖ We will solve problems together.</td>
</tr>
<tr>
<td>❖ Respect yourself.</td>
<td>❖ We will ask our group members for help before the teacher.</td>
</tr>
</tbody>
</table>

From *Caring School Community*, Developmental Studies Center, Devstu.org

Further Resources

Sample Math Class Norms

Classroom norms can be generated by the students themselves with facilitation by you as their teacher and fellow community member. They can also be discussed and innovated upon using a pre-determined list that you may have. Here are some sample math class norms you can use to generate discussion about class norms with your students.

- Errors are gifts that promote discussion.
- Answers are important, but they are not the math.
- Talk about each other’s thinking.
- Ask questions until ideas make sense.
- Use multiple strategies and multiple representations.
**Group Role Cards**

Collaborative groups benefit from clear roles for its members. Here are sample role cards that a group of four may find useful. On the next page is another set of typical group roles.

<table>
<thead>
<tr>
<th><strong>Questioner</strong></th>
<th><strong>Director</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Keep track of questions that the group has as you work on the math problem.</td>
<td>Make sure everyone is participating in the math work.</td>
</tr>
</tbody>
</table>
| ● *What* was your question?  
● *What do you mean by that?* | ● *What do you think?*  
● *What do you mean by that?* |

<table>
<thead>
<tr>
<th><strong>Illustrator</strong></th>
<th><strong>Connector</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Draw diagrams, tables, or other illustrations that show what the group is doing with the problem.</td>
<td>Make connections between what people in the group think and say.</td>
</tr>
</tbody>
</table>
| ● *Is this* what you were thinking?  
● *What do you mean by that?* | ● *How does this idea connect to that idea?*  
● *What do you mean by that?* |
Team Roles

Resource Manager:

- Call the teacher over when your team is stuck. Make sure that all questions are team questions.
  “What team question can we ask the teacher?”
  “Are we sure that no one here can answer the question?”
- Do not let your team stay stuck!

Facilitator:

- Make sure your team understands the entire task before you begin.
  “Who wants to read? Does everyone understand what we are being asked to do?”
  “What is the connection? How will it show in the graph? How will it show in the $x \rightarrow y$ table?”
- Keep your team together. Make sure everyone’s ideas are heard.
  “Are we all ready to move onto the next customer?”
  “Does anyone see it in a different way?”

Recorder/Reporter:

- Help your team organize a poster with your analysis. Your poster needs to show everyone’s ideas and be well organized. Use color, arrows, and other math tools to communicate your mathematics, reasons and connections.
  “How can we show that on the graph?”
  “How can we show that connection?”

Task Manager:

- Be sure that your team is accomplishing the task effectively and efficiently. Keep track of the time and tell the team when it is time to move forward to the next part of the task. Make sure that all talking is within your team and is helping you accomplish the task. Eliminate side conversations.
  “How can we divide the work most efficiently?”
  “We need to finish this part in 5 minutes, so we have time for…”

From College Preparatory Mathematics (CPM)
Classroom Organizational Tools
Math Notebooks

What are math notebooks?

Math notebooks are places for students to keep their math work in an organized fashion. They can be as simple as folders created from construction paper, composition books, or 3-ring binders with sections to organize materials into sections. These notebooks can include notes, vocabulary, solutions to investigation problems, homework, and responses to mathematical reflections/learning logs.

Why would I use math notebooks?

You might find it helpful for students to keep their work in an organized notebook. This allows the student to learn to organize their work as well as review past content. By reviewing your students' notebooks, you can get a clearer picture of their mathematical development. Notebooks also allow family members the opportunity to see the progress of their children in math class.

When do I use math notebooks?

In order for math notebooks to be effective, they should be used on a regular basis. Many of the SFUSD Math Core Curriculum units include lessons with loose pieces of paper, recording sheets, and math reflections/learning logs. Math notebooks can be used to organize all this information into one coherent record of math learning.

How do I set up math notebooks?

There many ways in which math notebooks can be set up. Some considerations are grade level, purpose of the notebooks, and classroom management style. Here is one example of how students might organize their math notebooks.

Section 1: Handouts
In this section keep assignment sheets, participation logs, and classroom rules and procedures.

Section 2: Journal
This section should include:

❖ Any and all work students do for in-class problems. This includes written, charts, pictures, or anything else to show their thinking.
❖ Any notes students take; including anything that will help them remember their thinking. They should also record notes from their group work. These notes are for reference as they solve in-class problems, answer homework questions, work on quizzes, and prepare for tests.
❖ Student’s learning log. This is a section for students to reflect on their learning through writing.

Section 3: Vocabulary
In this section, students will create mathematical descriptions with examples of words they need to know.

Section 4: Homework Assignments
This section includes homework assigned and also homework graded.
**Reviewing Math Notebooks**

You should check notebooks often during the first few weeks of school. It is important to give students feedback early to make sure notebooks are being used correctly and to address any problems. You should walk around the room while your students are working and give comments or suggestions on maintaining notebooks.

Since keeping notes in mathematics class might be new for some students, it is helpful to keep models of outstanding notes. This helps students understand your expectations. You can photocopy good examples to share with students. It might be helpful to have students evaluate their notes, journal entries, or vocabulary according to the models.

A Notebook Checklist can be used to evaluate students' notes periodically throughout a unit or at the end of a unit. Having students evaluate their notebooks before turning them in allows them to critically review their entries and organization.

You might choose to grade the journal, notes, or vocabulary sections of students' notebooks as well as the overall organization. Rubrics lend themselves nicely to the grading of notebooks, as you are generally looking for the completeness of ideas, notes, and vocabulary descriptions and records of the discussions from class. Consider giving "Credit," "Partial Credit," or "No Credit" as a grade for notebooks.

There are a variety of methods for checking student notebooks. Here are some ideas you might try:

- Read and respond to a few students' journal entries each day.
- Collect papers from students at the end of a lesson. Grade or respond to student work and then return the papers for students to replace in their notebooks.
- Collect notes at the end of a unit and grade them.
- Check notebooks at random.
- Give notebook quizzes. That is, periodically have students copy information from their notes on a sheet of paper; then grade just that information.
Sample Math Schedules

Each math class focuses on three aspects of learning: the social fabric of the classroom and learning community, the math practices used by the teacher and students, and the mathematical content.

Lower Elementary Sample Day

<table>
<thead>
<tr>
<th>Time</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opening</td>
<td>15 minutes In Opening Routines, the tone for the rest of the math class is set. Routines and math norms are introduced/reinforced. Time for Calendar or Math Talk.</td>
</tr>
<tr>
<td>Lesson Introduction</td>
<td>5 minutes Teacher introduces new content, a math task, or a new activity. See “Anticipating” under the “5 Practices for Orchestrating Productive Math Discussion” section.</td>
</tr>
<tr>
<td>Whole group work</td>
<td>15 minutes Students and the teacher work together to grapple with new concepts, procedures, problem-solving, and other content.</td>
</tr>
<tr>
<td>Group work</td>
<td>20 minutes Teacher can work with small groups during this time; students work in table or small groups, with partners, or independently; teacher can move through room making notes and taking anecdotal evidence of students’ understanding. See “Monitoring” under the “5 Practices for Orchestrating Productive Math Discussion” section. See “Rule of Four.”</td>
</tr>
<tr>
<td>Lesson close</td>
<td>10 minutes Students share what they have learned and questions they still may have. The teacher’s facilitation of the closure helps to bring out the key mathematics of the lesson, as well as revisiting the math norms of the lesson. See “Selecting, Sequencing, and Connecting” under the “5 Practices for Orchestrating Productive Math Discussion” section.</td>
</tr>
<tr>
<td>Closing Routines</td>
<td>10 minutes Closing Routines are a way for students to practice or review procedures and facts through games, exit slips, etc. This could also be a time for a Math Talk to close the math class (if it wasn’t part of opening the day’s lesson).</td>
</tr>
</tbody>
</table>
Sample Math Schedules

Upper Elementary / Secondary Sample Day

<table>
<thead>
<tr>
<th>Objectives</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SMP(s) Addressed</td>
<td></td>
</tr>
<tr>
<td>Norms</td>
<td></td>
</tr>
<tr>
<td>Materials</td>
<td></td>
</tr>
</tbody>
</table>

**Warm-up**
- Math talks (can be done at other times of the day as well)
- Mental math

**Launch**
- Whole group introduction to the lesson or task

See “Anticipating” under the “5 Practices for Orchestrating Productive Math Discussion” section.

**During**
- Group work (partners, triads, table groups)
- Center work

See “Monitoring” under the “5 Practices for Orchestrating Productive Math Discussion” section.

See “Rule of Four” and “Group Roles.”

**Closure/Extension**
- Whole group summary of math learned

See “Selecting, Sequencing, and Connecting” under the “5 Practices for Orchestrating Productive Math Discussion” section.

- Math games (as an extension)
Appendix:
Common Core State Standards for Mathematical Practice
Math Practice #1 – Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Math Practice #2 – Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Math Practice #3 – Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
Math Practice #4 – Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Math Practice #5 – Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Math Practice #6 – Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
Math Practice #7 – Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

Math Practice #8 – Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope 3, middle school students might abstract the equation $(y - 2)/(x-1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.