Part One: Purpose of the Validation Exam

There have been significant shifts in rigor, demanded by the Common Core State Standards (CCSS) in Mathematics. In the past, students have typically taken a course called “Algebra 1” in the 8th grade. There is a new course, with the same title, designed to be taken in 9th grade. There is not a simple, one-to-one correspondence between the previous California course entitled “Algebra 1”, and the new CCSS Algebra 1. Although there are some topics of overlap, the scope and rigor of these courses are significantly different. Many schools and districts have been transitioning from the old Algebra 1 course to the new Common Core Algebra 1 course, but efforts and implementation have varied greatly across the state. Because of this variation, many students have taken a course called “Algebra 1” in 8th grade and would like the opportunity to take Geometry in 9th grade in SFUSD. The district feels strongly that students who have taken a course covering the topics included in the Common Core Algebra 1 course should have the opportunity to take Geometry in 9th grade. However, many students have taken a course called “Algebra 1” that nonetheless did not address the scope and rigor expected of Common Core Algebra 1.

The primary purpose of the SFUSD Validation Exam is to provide a comprehensive check for understanding that matches the expected scope and rigor of the Common Core Algebra 1 course. The students not demonstrating mastery are highly recommended to take the 9th grade Common Core Algebra 1 course, because it would not be in the students’ best interest to skip over this essential mathematical content at the appropriate level of rigor demanded at SFUSD.

Part Two: Rationale for Using Mathematical Assessment Collaborative (MAC) Items

There are limited options for reliably assessing the balance of procedural fluency, conceptual understanding, and applications expected of students in CCSS Algebra 1. The depth of understanding must go beyond symbolic and procedural fluency and encompass a balance of conceptual knowledge, problem solving, mathematical modeling, and the construction of viable arguments. The district chose to use assessment items developed by the Silicon Valley Mathematics Initiative.

1 Updated 9/2018
The Silicon Valley Mathematics Initiative uses a performance assessment system as the cornerstone of the initiative. The math performance assessment system goes by the name Mathematics Assessment Collaborative (MAC). MAC began in 1998 and has grown from 23 districts to over 160 school districts and charter school networks located in several states. MAC is formed for the purpose of producing, scoring, and reporting student mathematical performance assessments at grades two through the first three years of college prep high school mathematics.

SFUSD feels strongly that the validation exam be comprised of items that have been thoroughly vetted by an extensive review and evaluation process. The steps of the review and evaluation process are as follows:

1. The original author of the task and rubric create a draft version.
2. The task is reviewed by peer experts in mathematics and education (Math Professors, K-12 Educators), revisions are suggested, and tasks are revised.
3. The task is piloted in multiple classrooms.
4. The student work is analyzed and revisions are made to the task to address access and learning concerns.
5. The task is field tested a second time and additional revisions are made.
6. The rubric is reviewed by an expert panel and revisions are made.
7. The student work is analyzed and student work is used to select the training and standardizing papers.
8. The tasks go through four editors before they are printed.
9. The task, rubric, and standardizing papers are finalized with a large panel of experts (12 to 20 members).
10. The tasks are administered to a minimum of 10,000 students across 30 - 70 school districts and the data is collected and used in the Tools for Teachers analysis.

It is our belief that these represent the best available assessment items for fulfilling the district’s purpose of validating students’ performance in an “Algebra 1” course to ensure they have the level of rigor necessary to be successful in future SFUSD mathematics courses.

Part Three: Structure of the Validation Exam

The validation exam is comprised of seven items. Each of these items can have multiple parts, typically an entry level “ramp” leading to a more cognitively demanding, and in some cases, open-ended question. These seven items have been selected to assess content areas deemed to be foundational for success in future mathematics courses. The content areas covered in a particular validation exam are comprised of a sample of the following important mathematical topics:

- Patterns and functions in the first and second degree
- Systems of linear equations
- Quadratic functions and parabolas
- Transformational geometry
- Coordinate geometry
- Bivariate data
- The Pythagorean Theorem
- Systems of linear inequalities
- Linear functions and graphs
- Linear equations in one or two variables
- Non-linear functions such as simple exponential functions

Note: The above list of topics is in no particular order or weighting.

Because this assessment is to validate mastery of the CCSS Algebra 1 (the course currently given in 9th grade in SFUSD), some of the topics address the more significant shifts. For example, Functions-Based Algebra and Statistics may be assessed through topics 1–3, 6, 9, and 11 in the list above. Furthermore, to support the goal of validating success in CCSS Geometry, topics from CCSS Grade 8 have been included, namely, Transformational and Coordinate Geometry and The Pythagorean Theorem (topics 4, 5, and 7 in the list above).

SFUSD provides details and released items from the Mathematics Validation Test on the district mathematics website: [http://www.sfusdmath.org/math-validation-test.html](http://www.sfusdmath.org/math-validation-test.html)

**Part Four: Mathematical Rigor**

The CCSS describe mathematical rigor as comprised of three components: procedural fluency, conceptual knowledge, and applications. Here is the description from the *Key Shifts in Mathematics* in the CCSS²:

**Rigor:** Pursue conceptual understanding, procedural skills and fluency, and application with equal intensity

Rigor refers to deep, authentic command of mathematical concepts, not making math harder or introducing topics at earlier grades. To help students meet the standards, educators will need to pursue, with equal intensity, three aspects of rigor in the major work of each grade: conceptual understanding, procedural skills and fluency, and application.

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Conceptual understanding: The standards call for conceptual understanding of key concepts, such as place value and ratios. Students must be able to access concepts from a number of perspectives in order to see math as more than a set of mnemonics or discrete procedures.

Procedural skills and fluency: The standards call for speed and accuracy in calculation. Students must practice core functions, such as single-digit multiplication, in order to have access to more complex concepts and procedures. Fluency must be addressed in the classroom or through supporting materials, as some students might require more practice than others.

Application: The standards call for students to use math in situations that require mathematical knowledge. Correctly applying mathematical knowledge depends on students having a solid conceptual understanding and procedural fluency.

The SFUSD MVT has been designed to provide opportunities for students to demonstrate evidence of mastery of rigorous mathematics as defined by the CCSS.

For example, one of the items involved analyzing data from Uber trips:

Sebastian is a new Uber driver. On his first day on the job, he picked up 11 different riders. Uber supplied him a log of each trip he drove that day, including the time it took to drive and the distance traveled for each trip. The table below is that data.

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1.5</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>1.5</td>
</tr>
<tr>
<td>8</td>
<td>2.5</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>13</td>
<td>4</td>
</tr>
<tr>
<td>16</td>
<td>5</td>
</tr>
<tr>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>20</td>
<td>5.5</td>
</tr>
<tr>
<td>23</td>
<td>6.5</td>
</tr>
</tbody>
</table>

Students are then asked a series of questions involving graphing the data and generating a linear model to estimate, describe, and predict relationships between time and distance.

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3 This is Item 4 on the July 2016 MVT
For example, Question 3 reads “Estimate the average speed for all trips and explain how you estimated the average speed”. This requires students to understand the concept of “average speed”, know what procedures could be used to perform the calculations, and be able to explain and justify his or her solution and approach.

Question 6 reads “On future days, Sebastian wants to make about $35 per hour from Uber fares. How much should he charge customers per mile?”. This question is a good example of an application, e.g. students apply the mathematics they know to solve meaningful problems in contexts generated from real life. Students must understand relationships between time, distance, and money in the situation and apply these to data to make decisions. These are important mathematical relationships and applications that provide students opportunities to demonstrate mastery of rigorous mathematics.

**Part Five: Establishing Cut Scores**

Ultimately, since the purpose of the Validation Exam is to provide students an opportunity to demonstrate mastery in the content and practices of CCSS Grade 8 and Algebra 1, there is a need to establish where the line between evidence of demonstrated mastery and lack of evidence of such mastery occurs. This line is called the “cut score”, and students scoring above the cut score are determined to have demonstrated mastery on the Validation Exam. The cut score for each Validation Exam is established by an expert panel comprised of national leaders including mathematicians, math university educators, teacher-leaders and K-12 educators. The process includes three basic analyses. The expert panel initially makes professional judgments of performance by comparing the task, its rubric and the Common Core State Standards of Mathematics to establish cut scores. Those preliminary scores are compared against a large random set of actual student work. Every score point is judged against student work. Comparison of statistical and historical score distributions are the final step in the process. This analysis produces a cut score for the Validation Exam.

Note, this process is called the Angoff method for establishing and defending cut-scores. This is a method described and supported in the *Standards for Educational and Psychological Testing* developed jointly by the National Council on Measurement in Education (NCME), American Education Research Association (AERA), and the American Psychological Association (APA).

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